PRINT YOUR NAME HERE:

HONOR CODE PLEDGE: "I have neither given nor received aid on this exam, nor have I concealed any violations of the honor code." Open book; SHOW ALL OF YOUR WORK!

SIGN YOUR NAME HERE:

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(30) 1. A fair (Pr[heads]=$\frac{1}{2}$) coin is flipped n times (independent flips), resulting in k heads. **Note:** Do **not** use the Demoivre-Laplace correction in this problem.

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(5) a. Compute mean $E[k]$ and variance σ_k^2 as functions of n.
(5) b. If $n = 1600$, compute $Pr[790 \leq k \leq 810]$ (give a specific number).
(5) c. If $n = 1600$, compute the largest b such that $Pr[k \geq b] \geq 0.9$.
(5) d. Compute a quadratic equation for the smallest n such that $Pr[k \geq 1000] \geq 0.95$.
(5) e. Solve the quadratic equation you computed in (d). What is n?
(5) f. Let $e = \frac{k}{n} - \frac{1}{2}$. Compute σ_e^2 as a function of n. Compute $\underset{n\to\infty}{\text{LIM}}\, \sigma_e^2$.

WRITE YOUR ANSWERS HERE:

(a): $E[k]$= σ_k^2= **(b):** Pr[]= **(c):** $b =$

(d): **(e):** n= **(f):** $\sigma_e^2 =$ $\underset{n\to\infty}{\text{LIM}}\, \sigma_e^2 =$

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(30) 2. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \right)$. $\left\{ \begin{array}{lccc} \text{Eigenvalues :} & 0 & 3 & 3 \\ \text{Eigenvectors :} & \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \end{array} \right\}$.

For (b) and (e), you may use $\mathcal{N}(\underline{\mu}, K)$ notation for multidimensional Gaussian pdfs.

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(5) a. If $x_1 = 2$ and $x_2 = 3$, compute x_3 (with probability one).
(5) b. Compute the joint marginal pdf $f_{x_1,x_3}(X_1, X_3)$.
(5) c. Let $y = x_1 + 2x_2 + 3x_3$. Compute σ_y^2 (give a number).
(5) d. Confirm *explicitly* that the Cauchy-Schwarz inequality holds for x_1 and x_2.
(5) e. If $z = x_1 - x_2$ and $w = x_1 + x_2 - 2x_3$, compute $f_{z|w}(Z|W)$.
(5) f. Compute $(\hat{x_1})_{LSE}(x_3)$, the least-squares estimate of x_1 based on x_3.

WRITE YOUR ANSWERS HERE:

(a): $x_3 =$ **(b):** $f_{x_1,x_3}(X_1, X_3) =$ **(c):** $\sigma_y^2 =$

(d): **(e):** $f_{z|w}(Z|W) =$ **(f):** $(\hat{x_1})_{LSE}(x_3) =$

#1:

#2:

#3:

$\sum$:

(40) 3. A coin with *unknown* $a = Pr[heads]$ is flipped N times, where N is known.
We observe r=#heads in N flips. Flips are independent of each other.

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(05) a. Compute $\hat{a}_{MLE}(R)$=maximum likelihood estimator of a based on R.

Now we are given the *a priori* pdf $f_a(A) = 10e^{-10A}, A > 0$; 0 otherwise.
Neglect $Pr[a > 1] = e^{-10} = 0.000045$ in (b) and (c).

(05) b. Compute a quadratic equation for $\hat{a}_{MAP}(R)$=maximum a posteriori probability estimator.

(05) c. If we observe $R = 20$ heads in $N = 92$ flips, compute $\hat{a}_{MAP}(20)$.

Now we are given *a priori* pdf $f_a(A) = \frac{1}{3}\delta(A - \frac{1}{4}) + \frac{2}{3}\delta(A - \frac{1}{2})$

(05) d. Compute an expression for $\hat{a}_{LSE}(R)$=least-squares estimator.

(05) e. If we observe $R = 2$ heads in $N = 3$ flips, compute $\hat{a}_{LSE}(2)$.

(10) f. Compute $E[a], E[r], E[ar], \lambda_{ar}$ for $N = 144$ using iterated expectation.
HINT: All but $E[a]$ come out to be positive integers. NOTE: $\sigma_r^2 = 321$.

(05) g. Compute an expression for $\hat{a}_{LLSE}(R)$=linear least-squares estimator.

WRITE YOUR ANSWERS HERE:

(a): $\hat{a}_{MLE} =$ **(b):** **(c):** $\hat{a}_{MAP}(20) =$

(d): $\hat{a}_{LSE} =$ **(e):** $\hat{a}_{LSE}(2) =$

(f): $E[a] =$ $E[r] =$ $E[ar] =$ $\lambda_{ar} =$ **(g):** $\hat{a}_{LLSE} =$
