PRINT YOUR NAME HERE:

HONOR CODE PLEDGE: "I have neither given nor received aid on this exam, nor have I concealed any violations of the honor code." Open book; SHOW ALL OF YOUR WORK!

SIGN YOUR NAME HERE:

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(40) 1. Random variables x, y have joint pdf $f_{x,y}(X,Y) = \begin{cases} cXY & \text{if } 0 < Y < X < 1 \\ 0 & \text{otherwise} \end{cases}$ where c is a constant. Random variable $z = y/x$.

(05) a. Compute the constant c in the pdf $f_{x,y}(X,Y)$.

(05) b. Are x and y independent? Explain your answer.

(05) c. Compute the marginal pdf $f_x(X)$.

(05) d. Compute the conditional pdf $f_{y|x}(Y|X)$ at $X = 1/2$.

(10) e. Compute the pdf $f_z(Z)$ using the *method of events*.

(10) f. Compute $Pr[(x + y) < 1]$. Hint: inner integral over y.

NOTE: Half-credit if you do this problem with "cXY" replaced with "c" in $f_{x,y}(X,Y)$.

WRITE YOUR ANSWERS HERE. SIMPLIFY TO A FRACTION.

(a): **(c):** **(e):**

(b): **(d):** **(f):**

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(40) 2. We flip coin A, which has Pr[heads]=2/3. All flips are independent.
If coin A lands heads, we flip coin B, which has Pr[heads]=3/4.
If coin A lands tails, we flip coin C, which has Pr[heads]=4/5.

(05) a. Compute Pr[the *second* coin flipped (whatever it is) lands heads].

(05) b. Compute Pr[Coin A landed heads|second coin flipped lands heads].

Now the second coin (whatever it is) is flipped $n-1$ more times (total of n flips).

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(05) c. Compute Pr[*All n flips* of the second coin (whatever it is) land heads].

(05) d. Compute Pr[Coin A landed heads|all n flips of second coin land heads].

(05) e. Compute $\lim_{n\to\infty}$ [your answer to (d)]. You don't need to be rigorous.

(05) f. Interpret your answer to (e): Explain why it (hopefully!) makes sense.

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(10) g. PROVE that if events E and F are independent, then events E and F' are also independent, where F'=set complement of F. HINT: Problem Set #1.

WRITE YOUR ANSWERS HERE. SIMPLIFY TO A FRACTION.

(a): **(c):**

(b): **(d):**

(e): **(f):**

(g):

(20) 3. For each of the following sets, answer each of the following 3 questions:
(i) Is it countable (C) or uncountable (U)? **(ii)** Is it a Borel set? Yes (Y) or No (N).
(iii) If a wheel of fortune is spun once, is Pr[(resulting number)$\in$(the set)]=0 or 1?
Circle your answer for each of the 4 sets:

NOTE: To eliminate guessing: right answers gain points; wrong answers **lose** points!
SCORES: (C vs. U):2 points each. (Y vs. N):1 point each. (0 vs. 1):2 points each.

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a. $\{rationals\}$	**(i):** C U	**(ii):** Y N	**(iii):** 0 1
b. $\{irrationals\}$	**(i):** C U	**(ii):** Y N	**(iii):** 0 1
c. $\{^{algebraic}_{irrationals}\}$	**(i):** C U	**(ii):** Y N	**(iii):** 0 1
d. $\{Cantor\ set\}$	**(i):** C U	**(ii):** Y N	**(iii):** 0 1

#1:

#2:

#3:

$\sum$: