1a. Setting $s=j\Omega$ gives $H_a(j\Omega)=\frac{1}{j\Omega}$. $|H_a(j0)|\to\infty$ and $|H_a(j\infty)|=0$.

1b. $H(z)=H_a(s=\frac{2}{2}\frac{z-1}{z+1})=\frac{z+1}{z-1}=\frac{Y(z)}{X(z)}\to y[n]-y[n-1]=x[n]+x[n-1]$.

1c. $H(e^{j\omega})=\frac{e^{j\omega}+1}{e^{j\omega}-1}=\frac{e^{j\omega/2}}{e^{j\omega/2}}\frac{e^{j\omega/2}+e^{-j\omega/2}}{e^{j\omega/2}-e^{-j\omega/2}}=\frac{2\cos(\omega/2)}{2j\sin(\omega/2)}=-j\frac{\cos(\omega/2)}{\sin(\omega/2)}=-j\cot(\omega/2)$.
$|H(e^{j0})|\to\infty$ and $H(e^{j\pi})=0$.

1d. For small ω=Ω, $H_a(j\Omega)=\frac{1}{j\Omega}$ and $H(e^{j\omega})\approx\frac{2}{j\omega}$.
Note that Ω becomes $\omega/2$. This is due to T=2.

2a. Setting $s=j\Omega$ gives $H_a(j\Omega)=\frac{a}{j\Omega+a}$. $|H_a(j0)|=1$ and $|H_a(j\infty)|=0$.

2b. $H(z)=H_a(s=\frac{2}{2}\frac{z-1}{z+1})=a/[\frac{z-1}{z+1}+a]=\frac{a(z+1)}{(a+1)z+(a-1)}$.

2c. $H(e^{j\omega})=\frac{a(e^{j\omega}+1)}{(a+1)e^{j\omega}+(a-1)}=\frac{e^{j\omega/2}}{e^{j\omega/2}}\frac{a(e^{j\omega/2}+e^{-j\omega/2})}{a(e^{j\omega/2}+e^{-j\omega/2})+(e^{j\omega/2}-e^{-j\omega/2})}$ becomes
$H(e^{j\omega})=\frac{a\cos(\omega/2)}{a\cos(\omega/2)+j\sin(\omega/2)}$. $H(e^{j0})=\frac{a}{a+0}=1$. $H(e^{j\pi})=\frac{0}{0+j}=0$.

2d. For small ω=Ω, $H_a(j\Omega)=\frac{a}{j\Omega+a}$ and $H(e^{j\omega})\approx\frac{a}{a+j\sin(\omega/2)}=\frac{a}{j\omega/2+a}$.
Note that Ω becomes $\omega/2$. This is due to T=2.

2e. $H(z)=H_a(s=\frac{2}{2}\frac{z-1}{z+1})=b/[\frac{z-1}{z+1}+b]=\frac{b(z+1)}{(b+1)z+(b-1)}=\frac{\frac{b}{b+1}(z+1)}{z+\frac{b-1}{b+1}}$.
$H(z)$ has a pole at $-a$ if $a=\frac{b-1}{b+1}\to b=\frac{1+a}{1-a}$.

3a. $\begin{cases} h_{\text{IDEAL}}[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}(j\omega)e^{j\omega n}d\omega \\ \text{Then } h[n]=h_{\text{IDEAL}}[n]w[n]\end{cases}=\begin{cases}(-1)^n/n & \text{for } n\neq 0\\ 0 & \text{for } n=0\end{cases}$.

```
H1=[((-1).^[-10:-1])./[-10:-1] 0 ((-1).^[1:10])./[1:10]];
W=0.54+0.46*cos(2*pi*[-10:10]/21);plot(abs(fft(H1.*W,256)))
```

3b. $h[n]=\text{IDFT}\{\begin{cases} j\omega & \text{for } \omega=\frac{2\pi k}{21}, 0\le k\le 10\\ j(\omega-2\pi) & \text{for } \omega=\frac{2\pi k}{21}, 11\le k\le 20\end{cases}$

```
H2=real(ifft([j*2*pi*[0:10]/21 j*2*pi*[11:20]/21-j*2*pi]));
plot(abs(fft([H2(12:21) H2(1:11)],256)))
```

---

4. `firpm(59,[0 0.2 0.3 1],[0 1.0 0 0],'differentiator')`.
`'differentiator'` uses weight $W(e^{j\omega})=\frac{1}{\omega}$; `'hilbert'` uses $W(e^{j\omega})=1$.

5a. `plot(Y),plot(abs(fft(Y)))` See next page. Actually, `Y` came from
`Y=filter(1,[1 0 0 0 0 0 0 0 0.99],cos([1:1024].^2/1000));`

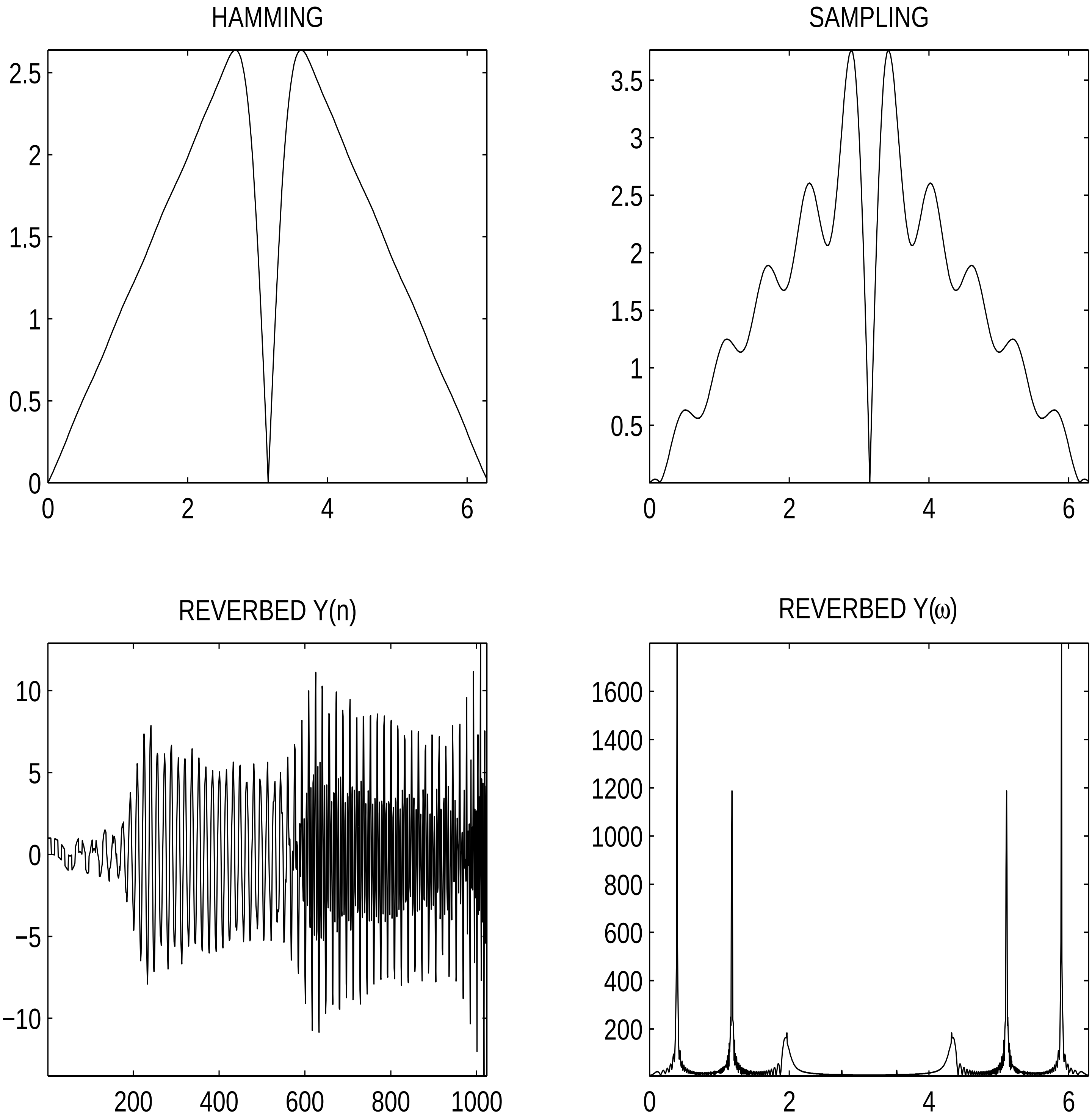
5b. 8 equispaced spikes→ 8 poles near $|z|$=1: $\{.99^{\frac{1}{8}}e^{j\frac{2\pi k+\pi}{8}}, 0\le k\le 7\}$.
N=7. Two spikes are very small, but they are still there (zoom in).

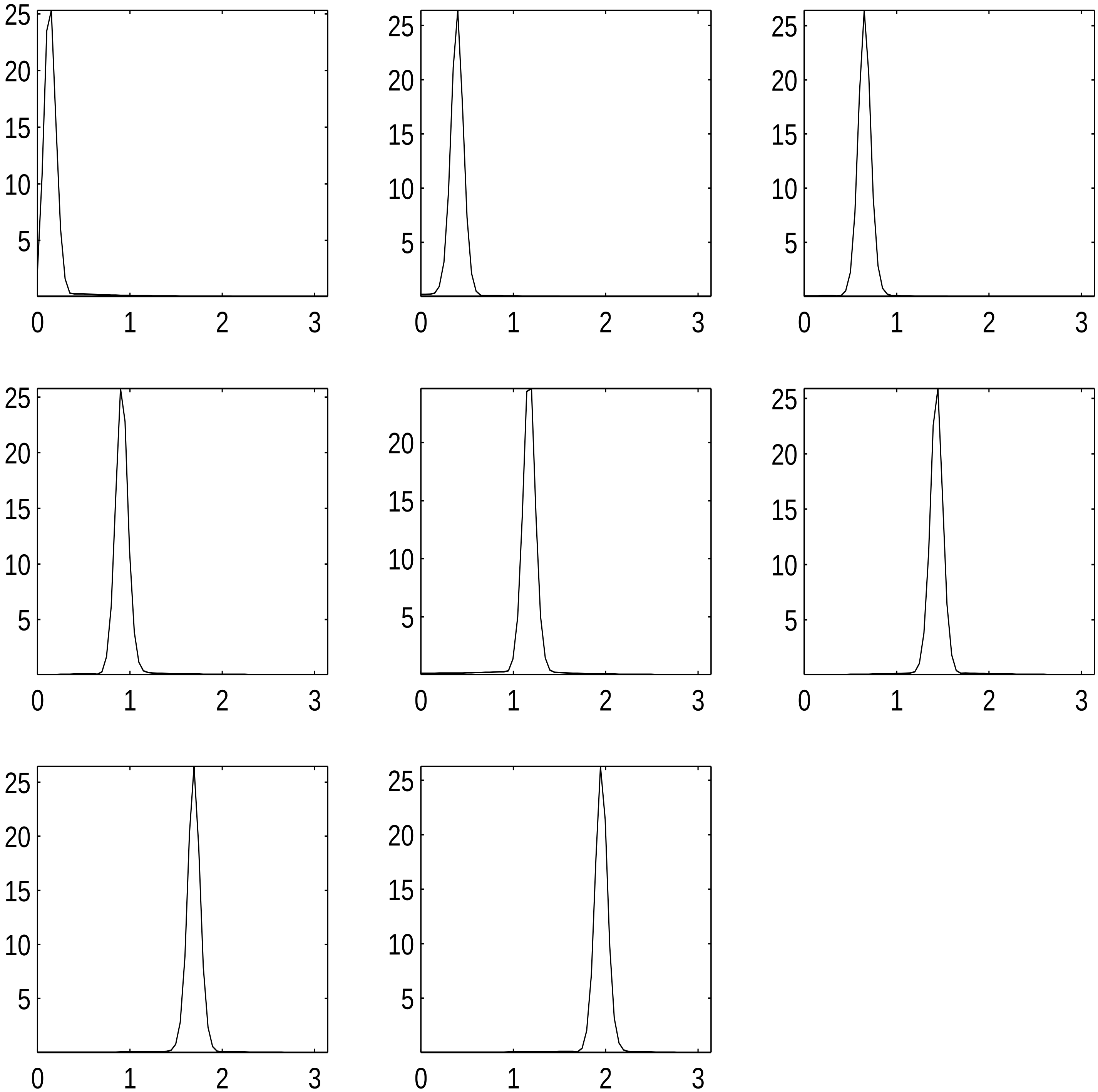
5c. $H(z)=\frac{z^8}{z^8+0.99}=\frac{Y(z)}{X(z)}$ →y[n]+0.99y[n–8]=x[n]. Use to get x[n] from y[n].
This is *blind deconvolution:* inferring both h[n] and x[n] from y[n]=h[n]*x[n].

---

6. `cos([1:1024].^2/1000)` is a *chirp*; its frequency increases with time.

```
X=cos([1:1024].^2/1000);W=0.54+0.46*cos(2*pi*[-64:63]/128);
for I=1:8;F=abs(fft(X((I-1)*128+[1:128]).*W));subplot(33I),
plot(linspace(0,pi,64),F(1:64)),axis tight;end
```


HAMMING
SAMPLING
REVERBED Y(n)
REVERBED Y(ω)


25 20 15 10 5
0 1 2 3
25 20 15 10 5
0 1 2 3
25 20 15 10 5
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25 20 15 10 5
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20 15 10 5
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