1a. **Reverse** the second sequence: $\{4,3,2,2\} \to \{2,2,3,4\}$, repeat the cycle, and shift:

y(0): $\begin{smallmatrix}\{1,2,3,1,\underline{1},2,3,1\}\\ \{4,2,2,3,\underline{4},2,2,3\}\end{smallmatrix} \to (1)(4)+(2)(2)+(3)(2)+(1)(3)=17.$

y(1): $\begin{smallmatrix}\{1,2,3,1,\underline{1},2,3,1\}\\ \{3,4,2,2,\underline{3},4,2,2\}\end{smallmatrix} \to (1)(3)+(2)(4)+(3)(2)+(1)(2)=19.$

y(2): $\begin{smallmatrix}\{1,2,3,1,\underline{1},2,3,1\}\\ \{2,3,4,2,\underline{2},3,4,2\}\end{smallmatrix} \to (1)(2)+(2)(3)+(3)(4)+(1)(2)=22.$

y(3): $\begin{smallmatrix}\{1,2,3,1,\underline{1},2,3,1\}\\ \{2,2,3,4,\underline{2},2,3,4\}\end{smallmatrix} \to (1)(2)+(2)(2)+(3)(3)+(1)(4)=19.$ $\{\underline{17},19,22,19\}$.

1b. $X_1(0)=1+2+3+1=07.$ $X_1(1)=1+2(-j)+3(-1)+1(j)=-2-j.$

$X_1(2)=1-2+3-1=01.$ $X_1(3)=1+2(j)+3(-1)+1(-j)=-2+j.$

$X_2(0)=4+3+2+2=11.$ $X_2(1)=4+3(-j)+2(-1)+2(j)=+2-j.$

$X_2(2)=4-3+2-2=01.$ $X_2(3)=4+3(j)+2(-1)+2(-j)=+2+j.$

Multiply: $Y(0)=(7)(11)=77.$ $Y(1)=(-2-j)(2-j)=-5.$ $Y(2)=(1)(1)=1.$

Multiply: $Y(3)=(-2+j)(2+j)=-5$ or conjugate symmetry: $Y(3)=Y(1)^*=-5.$

$y(0)=\frac{1}{4}[77-5+1-5]=17.$ $y(1)=\frac{1}{4}[77-5(j)+1(-1)-5(-j)]=19.$
$y(2)=\frac{1}{4}[77+5+1+5]=22.$ $y(3)=\frac{1}{4}[77-5(-j)+1(-1)-5(j)]=19.$

2a. $X(z)=1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}$. $X_k=1+e^{-j\frac{2\pi k}{4}}+e^{-j\frac{4\pi k}{4}}+e^{-j\frac{6\pi k}{4}}+e^{-j\frac{8\pi k}{4}}+e^{-j\frac{10\pi k}{4}}$.

2b. Using $e^{-j\frac{8\pi k}{4}}=1$ and $e^{-j\frac{10\pi k}{4}}=e^{-j\frac{2\pi k}{4}}$, X_k becomes $X_k=2+2e^{-j\frac{2\pi k}{4}}+e^{-j\frac{4\pi k}{4}}+e^{-j\frac{6\pi k}{4}}$.

2c. From $X_k=\sum_{n=0}^{3} x[n]e^{-j2\pi nk/4}$, we can read off $x_{\text{NEW}}[n]=\{\underline{2},2,1,1\}$.

2d. $x[n]=\{\underline{1},1,1,1,1,1\}$ becomes *aliased* to $\{\underline{2},2,1,1\}$.

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3. `FX1=fft(X1)` has the following nonzero values in its first half (neglecting roundoff):

FX1(565) $=6000+j8000=10000e^{j53^o} \to 2\frac{10000}{4000}\cos((565-1)\frac{1000}{4000}t+53^o)=\boxed{5\cos(2\pi141t+53^o)}$

FX1(1085) $=2000+j2000=2828e^{j45^o} \to 2\frac{2828}{4000}\cos((1085-1)\frac{1000}{4000}t+45^o)=\boxed{\sqrt{2}\cos(2\pi271t+45^o)}$

FX1(1257) $=10000+j24000=26000e^{j67^o} \to 2\frac{26000}{4000}\cos((1257-1)\frac{1000}{4000}t+67^o)=\boxed{13\cos(2\pi314t+67^o)}$

FX1(1461) $=14000+j48000=50000e^{j73^o} \to 2\frac{50000}{4000}\cos((1461-1)\frac{1000}{4000}t+73^o)=\boxed{25\cos(2\pi365t+73^o)}$

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4. `FX2=fft(X2);FX2(24002:48000)=0;Y=real(ifft(FX2));` `Y` has the noise eliminated.

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5. `Y=real(ifft(fft(X3,144000)./fft(H,144000)));` Handel's "Hallelujah Chorus."