

1a. $x[n]$ real & even $\rightarrow x_k$ real & even. $x_k = \frac{1}{6} \sum_{n=0}^5 x[n] e^{-j2\pi nk/6} = \frac{1}{6} \sum_{n=0}^3 2 \cos(2\pi nk/6)$.
 $x_0 = \frac{1}{6}(18+12+6+0+6+12) = 9$. $x_1 = \frac{1}{6}(18+12(2)\cos(2\pi/6)+6(2)\cos(4\pi/6)) = 4$.
 $x_3 = \frac{1}{6}(18-12+6-0+6-12) = 1$. $x_2 = \frac{1}{6}(18+12(2)\cos(4\pi/6)+6(2)\cos(8\pi/6)) = 0$.
 $x_k = \boxed{9, 4, 0, 1, 0, 4}$. Check: `fft([18,12,6,0,6,12])/6` gives `9,4,0,1,0,4`.

1b,c. $\frac{1}{6}(18^2 + 12^2 + 6^2 + 0^2 + 6^2 + 12^2) = (9^2 + 4^2 + 0^2 + 1^2 + 0^2 + 4^2) = \boxed{114}$.

$$2. \cos\left(\frac{\pi k}{4}\right) + \sin\left(\frac{3\pi k}{4}\right) = \frac{1}{2}e^{j\frac{2\pi k}{8}} + \frac{j}{2}e^{j\frac{2\pi 3k}{8}} - \frac{j}{2}e^{-j\frac{2\pi 3k}{8}} + \frac{1}{2}e^{-j\frac{2\pi k}{8}} = \frac{1}{2}e^{j\frac{2\pi k}{8}} + \frac{j}{2}e^{j\frac{2\pi 3k}{8}} - \frac{j}{2}e^{j\frac{2\pi 5k}{8}} + \frac{1}{2}e^{j\frac{2\pi 7k}{8}}$$

2a. DTFS coefficients x_k have period $8=N \rightarrow x[n]$ has period $N=8$.

2b. $x_{\text{EVEN}}[n] = \boxed{0, 4, 0, 0, 0, 0, 0, 4}$. (2c) $x_{\text{ODD}}[n] = \boxed{0, 0, 0, 4j, 0, -4j, 0, 0}$.

2d. Average power = $\frac{2}{8}(0^2 + 4^2 + 0^2 + |4j|^2) = 2(1^2 + (\sqrt{2})^2 + (-1)^2 + 0^2) = \boxed{8}$.

3a. $e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega} = \boxed{1 + 2\cos(\omega) + 2\cos(2\omega)}$.

3b. $3e^{j\omega} + 2 + e^{-j\omega} = \boxed{2 + 4\cos(\omega) + 2j\sin(\omega)}$.

4a. $\frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}} = \text{DTFT}\left\{\left(\frac{1}{2}\right)^n u[n]\right\}$. $\left(\frac{1}{2}\right)^3 = \boxed{\frac{1}{8}}$.

4b. $2\cos(3\omega) = e^{j3\omega} + e^{-j3\omega} = \text{DTFT}\{1, 0, 0, 0, 0, 0, 1\}$. At $n=3$: $\boxed{1}$.

5a. $X(e^{j\pi}) = 1 - 4 + 3 - 2 + 5 - 7 + (-45) - 7 + 5 - 2 + 3 - 4 + 1 = \boxed{-53}$.

5b. Since $45 > 2(1+4+3+2+5+7) = 44$, $\arg[X(e^{j\omega})] = \boxed{-\pi}$. (5c) $2\pi x[0] = \boxed{-90\pi}$.

5d. $2\pi[(-45)^2 + 2(1^2 + 4^2 + 3^2 + 2^2 + 5^2 + 7^2)] = \boxed{4466\pi}$.

6a. It is actually "Matlab-Breakfast of Champions" *scrambled*. Hard to interpret it.

6b. Looking carefully, you can see that I $\boxed{\text{flipped the spectrum}}$.

6c. `Y=real(ifft(fftshift(fft(X))))` flips the spectrum back.

6d. Comparing X and Y, the latter is the former with every other value's sign changed.

This follows from the modulation property: $\text{DTFT}\{(-1)^n x[n]\} = X(e^{j(\omega+\pi)})$.

