

1a. $Y(z) - 5(z^{-1}Y(z) + y[-1]) + 6(z^{-2}Y(z) + z^{-1}y[-1] + y[-2]) = \frac{4z}{z-1}$. Substitute $y[-1] = y[-2] = 1$:

$$Y(z)(1 - 5z^{-1} + 6z^{-2}) = \frac{4z}{z-1} - [(6-5) + 6z^{-1}] \frac{z-1}{z-1} \rightarrow Y(z) = \frac{3z^3 - 5z^2 + 6z}{(z-2)(z-3)(z-1)}$$

$$\frac{Y(z)}{z} = \frac{3z^2 - 5z + 6}{(z-1)(z-2)(z-3)} = \frac{2}{z-1} - \frac{8}{z-2} + \frac{9}{z-3} \rightarrow y[n] = \boxed{[2 - 8(2)^n + 9(3)^n]u[n]} = \{3, 13, 51, 181, 603, \dots\}$$

1b. $Y(1)=1; Y(2)=1; \text{for } I=3:7; Y(I)=4+5*Y(I-1)-6*Y(I-2); \text{end}; Y \rightarrow \text{same answer.}$

2a. $Y(z) = \frac{2z}{z-1} \frac{z+2}{z+2} + \frac{z}{z+2} \frac{z-1}{z-1} = \frac{3z^2+3z}{(z-1)(z+2)}$. $X(z) = \frac{z}{z-1}$. $H(z) = \frac{Y(z)}{X(z)} = \frac{3z^2+3z}{(z-1)(z+2)} \frac{z-1}{z} = \boxed{3 \frac{z+1}{z+2}}$.

2b. ZEROS: -1 . POLES: -2 . (2c) $h[n] = \boxed{3(-2)^n u[n] + 3(-2)^{n-1} u[n-1]}$.

2d. $\frac{Y(z)}{X(z)} = H(z) = 3 \frac{z+1}{z+2}$. Cross-multiply $\rightarrow \boxed{y[n] + 2y[n-1] = 3x[n] + 3x[n-1]}$.

3a. $6 = H(0) = C \frac{(0-3)(0-4)}{(0-1)(0-2)} \rightarrow C = 1$. $H(z) = \frac{z^2 - 7z + 12}{z^2 - 3z + 2}$.

3b. $Y(z) = H(z)X(z) = \frac{z^2 - 7z + 12}{z^2 - 3z + 2} \frac{z^2 - 3z + 2}{z^2} = \frac{z^2 - 7z + 12}{z^2} \rightarrow \boxed{\{1, -7, 12\}}$

3c. $\frac{H(z)}{z} = \frac{z^2 - 7z + 12}{z(z-1)(z-2)} = \frac{6}{z} - \frac{6}{z-1} + \frac{1}{z-2} \rightarrow h[n] = \boxed{6\delta[n] - 6u[n] + (2)^n u[n]}$.

3d. $\frac{Y(z)}{X(z)} = H(z) = \frac{z^2 - 7z + 12}{z^2 - 3z + 2} \rightarrow \boxed{y[n] - 3y[n-1] + 2y[n-2] = x[n] - 7x[n-1] + 12x[n-2]}$.

4a. Because this difference equation is unstable, since poles $\{3, 4\}$ outside the unit circle.

4b. $G(z) = \frac{1}{H(z)} = \frac{z^2}{(z-3)(z-4)}$. $\frac{G(z)}{z} = \frac{4}{z-4} - \frac{3}{z-3} \rightarrow g[n] = \boxed{3(3)^n u[-n-1] - 4(4)^n u[-n-1]}$.

4c. $g[n] = \{\dots 3^{-9} - 4^{-9}, 3^{-8} - 4^{-8}, \dots 3^{-1} - 4^{-1}, 0, 0\}$. Delay this by 10.

$N = -10 : -1; G = 3 \cdot \widehat{(N+1)} - 4 \cdot \widehat{(N+1)}; \text{conv}(G, [1 \ -7 \ 12])$. Output ≈ 0 except at index = 11.

5a. A reverbed "Matlab-Breakfast of Champions" (sounds like a science fiction film).

5b. Z-transform: $Y(z) = X(z)(1 + (0.8)z^{-3(1024)} + (0.8)^2 z^{-6(1024)} + (0.8)^3 z^{-9(1024)} + \dots)$

$$= \frac{X(z)}{1 - 0.8z^{-3072}} \rightarrow H(z) = \frac{1}{1 - 0.8z^{-3072}}$$
. Minimum phase \rightarrow stable & causal $g[n]$ exists.

5c. $G(z) = \frac{1}{H(z)} = 1 - 0.8z^{-3072} \rightarrow \boxed{x[n] = y[n] - 0.8y[n-3072]}$. Obvious once you see it.

5d. $Y = \text{filter}([1 \ \text{zeros}(1, 3071) \ -0.8], [1], X)$. 3 nonzero numbers (two are ones).

X is from $X = \text{filter}([1], [1 \ \text{zeros}(1, 3071) \ -0.8], X1)$; $X1$ is from `p1.mat`.