1a. $\boxed{z^2+3z+4}$; ROC: $\boxed{0 \le z < \infty}$. (1b) $\frac{z}{z-2}\frac{z-1}{z-1}+\frac{z}{z-1}\frac{z-2}{z-2}=\boxed{\frac{2z^2-3z}{z^2-3z+2}}$. ROC: $\boxed{|z|>2}$.

1c. $\frac{z}{z-\frac{1}{3}}\frac{z-2}{z-2}-\frac{z}{z-2}\frac{z-\frac{1}{3}}{z-\frac{1}{3}}=\boxed{\frac{-5z/3}{z^2-\frac{7}{3}z+\frac{2}{3}}}$. ROC: $\boxed{\frac{1}{3}<|z|<2}$.

1d. $\boxed{\text{DOES NOT EXIST}}$ since ROC is $3<|z|<\frac{1}{2}=\emptyset$!

2a. $\frac{z+1}{2z}=\frac{1}{2}+\frac{1}{2}z^{-1}\rightarrow\boxed{\{\underline{\frac{1}{2}},\frac{1}{2}\}}$. Students always seem to have trouble.

2b. $\frac{z-1}{z-2}=\frac{z-2}{z-2}+\frac{1}{z-2}\rightarrow\boxed{\delta[n]+2^{n-1}u[n-1]}$. OR: $\frac{z}{z-2}-\frac{1}{z-2}\rightarrow\boxed{2^n u[n]-2^{n-1}u[n-1]}$.

2c. $\frac{2z+3}{z^2(z+1)}=\frac{1}{z^2}\frac{2z+3}{z}\frac{z}{z+1}\rightarrow\boxed{2(-1)^{n-2}u[n-2]+3(-1)^{n-3}u[n-3]}$.

2d. $\frac{1}{z}\frac{z^2+3z}{z^2+3z+2}=\frac{z+3}{(z+1)(z+2)}=\frac{2}{z+1}-\frac{1}{z+2}\rightarrow\boxed{2(-1)^n u[n]-(-2)^n u[n]}$.

2e. $\frac{1}{z}\frac{z^2-z}{z^2-2z+2}=\frac{z-1}{(z-(1+j))(z-(1-j))}=\frac{1/2}{z-(1+j)}+\frac{1/2}{z-(1-j)}\rightarrow\boxed{(\sqrt{2})^n\cos(\frac{\pi}{4}n)u[n]}$.

3a. $\boxed{4u[n]+5(2)^n u[n]-6(3)^n u[-n-1]}$. $2<|z|\rightarrow$1,2 causal and $|z|<3\rightarrow$3 anticausal.

3b. $\boxed{-2\sqrt{2}(5)^n\cos(0.91n+45^o)u[-n-1]}$. 1+j=$\sqrt{2}e^{j45^o}$ and 3+4j=$5e^{j0.91}$ and $5>1$.

3c. $\boxed{10(\sqrt{2})^n\cos(\frac{\pi}{4}n+53^o)u[n]-2\sqrt{2}(5)^n\cos(0.91n+45^o)u[-n-1]}$.

$\sqrt{2}<|z|\rightarrow 1^{st}$ term causal & unstable. $|z|<5\rightarrow 2^{nd}$ term anticausal & stable.

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4. 1^{st} **term:** $\frac{-5z/3}{z^2-\frac{7}{3}z+\frac{2}{3}}$ from #1c. 2^{nd} **term:** $\frac{z}{z-\frac{1}{2}}$. Partial fraction expansion (Matlab):

$\frac{1}{z}\frac{-5z^2/3}{(z-\frac{1}{2})(z-\frac{1}{3})(z-2)}=\frac{10/3}{z-\frac{1}{2}}-\frac{2}{z-\frac{1}{3}}-\frac{4/3}{z-2}$. $\boxed{\frac{10}{3}(\frac{1}{2})^n u[n]-2(\frac{1}{3})^n u[n]+\frac{4}{3}(2)^n u[-n-1]}$.

Matlab: `A=conv([1 -7/3 2/3],[1 -1/2]);B=[-5/3 0];[R P]=residue(B,A);`

Result: `R=[-4/3 10/3 -2] P=[2 1/2 1/3]`. Can now just read off the answer.

Check: Compare `conv([2.^[-10:-1] (1/3).^[0:10]],(1/2).^[0:10])` with

`[(4/3)*2.^[-10:-1] (10/3)*(1/2).^[0:10]-2*(1/3).^[0:10]]`. Agree closely.

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5. Stable if $\{|z|=1\}\subset$ROC. Causal if ROC=$\{|z|>a\}$. Anticausal if ROC=$\{|z|<b\}$.

Two-sided if ROC=$\{a<|z|<b\}$.

(a) $\boxed{\text{2-sided, unstable}}$ (b) $\boxed{\text{2-sided, unstable}}$ (c) $\boxed{\text{causal, stable}}$ (d) $\boxed{\text{anticausal, stable}}$

(e) $\boxed{\text{2-sided, stable}}$. Note that giving you X(z) would have been a red herring!