

1. Use the formulae  $T = \frac{2\pi}{\omega}$  (continuous time) and  $\frac{2\pi}{\omega} = \frac{N}{D} \rightarrow \text{period} = N$  (discrete time).

(a)  $\frac{2\pi}{0.16\pi} = \frac{200}{16} = \boxed{12.5}$  (b)  $\frac{200}{16} = \frac{25}{2} \rightarrow N = \boxed{25}$  (c) no  $\pi \rightarrow \boxed{\infty}$

(d)  $\frac{2\pi}{0.16\pi} = \frac{25}{2} \rightarrow 25$ .  $\frac{2\pi}{0.15\pi} = \frac{40}{3} \rightarrow 40$ .  $\text{LCM}[25, 40] = \boxed{200}$

2.  $x[n] = x(t = n/1000)$ ; alias  $x[n]$  using  $\cos((\pi + \omega_o)n + \theta) = \cos((\pi - \omega_o)n - \theta)$ .

(a)  $x[n] = \cos(2\pi \frac{30}{100}n) + \cos(2\pi \frac{70}{100}n + \pi) = \cos(0.6\pi n) + \cos(1.4\pi n + \pi)$ . Alias the  $2^{\text{nd}}$  term:  
 $\cos(1.4\pi n + \pi) = \cos(0.6\pi n - \pi) = -\cos(0.6\pi n) \rightarrow \text{output} = 0!$  (b): See below.

3.  $x[n] = x(t = n/1000)$ ; alias  $x[n]$  using  $\cos((\pi + \omega_o)n + \theta) = \cos((\pi - \omega_o)n - \theta)$ .

(a)  $x[n] = \sin(2\pi \frac{30}{100}n) + \sin(2\pi \frac{70}{100}n) = \sin(0.6\pi n) + \sin(1.4\pi n)$ . Alias the  $2^{\text{nd}}$  term:  
 $\sin(1.4\pi n) = \cos(1.4\pi n - \frac{\pi}{2}) = \cos(0.6\pi n + \frac{\pi}{2}) = -\sin(0.6\pi n) \rightarrow \text{output} = 0!$  (b): See below.

4a.  $3 - 4j = \boxed{5e^{-j0.927}}$  (4b)  $12 + 5j = \boxed{13e^{j0.395}}$

4c.  $[(3 - 4j) + (12 - 5j)]^2 = (15 - 9j)^2 = 225 - 81 - 2(j15)9 = 144 - 270j = \boxed{306e^{-j1.081}}$

4d.  $\frac{3 - 4j}{12 + 5j} = \frac{5e^{-j0.927}}{13e^{j0.395}} = \frac{5}{13}e^{j(-.927 - .395)} = \boxed{0.385e^{-j1.322}}$

OR:  $\frac{3 - 4j}{12 + 5j} \frac{12 - 5j}{12 - 5j} = \frac{(36 - 20) - j(48 + 15)}{5^2 + 12^2} = .0947 - .373j = 0.385e^{-j1.322}$  (ugh).

5a.  $4e^{jt} + 4e^{-jt} = 8 \cos(t)$ . (5b)  $-je^{j2t} + je^{-j2t} = e^{-j\frac{\pi}{2}}e^{j2t} + e^{j\frac{\pi}{2}}e^{-j2t} = 2 \cos(2t - \frac{\pi}{2}) = \boxed{2 \sin(2t)}$

5c.  $je^{j(3t+1)} - je^{-j(3t+1)} = e^{j(\frac{\pi}{2}+1)}e^{j3t} + e^{-j(\frac{\pi}{2}+1)}e^{-j3t} = 2 \cos(3t + \frac{\pi}{2} + 1) = \boxed{-2 \sin(3t + 1)}$

5d.  $(3 + j4)e^{j6t} + (3 - j4)e^{-j6t} = 5e^{j.927}e^{j6t} + 5e^{-j.927}e^{-j6t} = \boxed{10 \cos(6t + .927)}$

6a. X is voice: "Matlab-Breakfast of Champions." Spectrum is bandlimited to 8 kHz.

6b. Y is the same voice speeded up by a factor of two. Spectrum is now aliased slightly.

6c. Z slowed down has a slight lisp—this is aliasing. Spectrum is now very aliased!

3 plots of spectra produced from Matlab code given in the problem set are below.

Note the high frequency components are different in each plot, due to aliasing.

**(2b:)** Upper left spectrum (cosines). **(3b:)** Lower left spectrum (sines; mult. by  $j$ ).

