Review Session 1 - EECS 451, Winter 2009

Feb 11th, 2009

OUTLINE

- Review of important concepts (Chapters 1-3)
- Exercises (Homework $1-4$)

Concepts from Chapter 1-2

1. Sampling theorem: If a continuous signal $x_a(t)$ is bandlimited (i.e. $X_a(F) = 0\ \forall |F| > F_{max}$), the signal can be reconstructed back exactly from its samples
   - if the sampling rate is greater than the *Nyquist rate* ($2F_{max}$), i.e. $F_s \geq 2F_{max}$
   - otherwise, we have aliasing
2. Classification of general DT systems: For $y(n) = \mathcal{T}(x(n))$, the system is
   - memoryless if $y(n)$ depends only on the present input
   - causal if $y(n)$ does not depend on the future inputs
   - time-invariant if $\mathcal{T}(x(n-k)) = y(n-k)$
   - linear if $\mathcal{T}(a_1x_1(n) + a_2x_2(n)) = a_1\mathcal{T}(x_1(n)) + a_2\mathcal{T}(x_2(n))$
   - BIBO stable if every bounded input produces a bounded output
3. Impulse response
   - *Response of the system to an impulse input*, i.e. $h(n) := \mathcal{T}(\delta(n))$
   - Impulse response can be defined for any system
4. Linear Time Invariant (LTI) system
   - Is completely characterized by its impulse response $h(n)$
   - The output $y(n)$ is given by $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$
   - Is causal iff $h(n) = 0\ \forall n < 0$
   - Is BIBO stable iff $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$
5. Classification of LTI DT systems by "system function $H(z)$" ($h(n) \overset{Z}{\leftrightarrow} H(z)$). The system is

- causal iff the ROC is the exterior of a circle of radius $r < \infty$ and includes $z = \infty$
- BIBO stable iff the ROC includes the unit circle

6. The system defined by linear constant coefficient difference equation $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$
   - is LTI and causal
   - is called a Moving average (MA) system if $N = 0$ and $M > 0$
   - is called an Auto Regressive (AR) system if $N > 0$ and $M = 0$
   - is called an Auto Regressive Moving Average (ARMA) system if $N > 0$ and $M > 0$
   - has a rational form $H(z)$

Concepts from Chapter 3

1. 2-sided z-transforms: For a given DT signal $x(n)$
   - $X(z) := \sum_{n=-\infty}^{\infty} x(n) z^{-n}$, where z is complex valued
   - z transform is linear
2. ROC (Region of Convergence)
   - The set of values of z for which the sequence $x(n)z^{-n}$ is absolutely summable, i.e. $\{z \in \mathbf{C} : \sum_{n=-\infty}^{\infty} |x(n)z^{-n}| < \infty\}$, where $\mathbf{C}$ is the set of complex numbers.
   - Simply put, ROC indicates the region of z where $X(z)$ is finite.
   - By definition, ROC cannot contain any poles.
3. The shape of ROCs
   - The ROC of an anti-causal signal is of the form $|z| < |a|$.
   - The ROC of a causal signal is of the form $|z| > |a|$.
   - The ROC of a two sided signal is of the form $|a| < |z| < |b|$.
   - The ROC of a finite length signal is the entire $z-$space except for $z = 0$ and/or $z = \infty$.
4. Useful z-transformation pairs
   - If $x(n) = a^n u(n)$, then $X(z) = \frac{z}{z-a}$, ROC $= |z| > |a|$.
   - If $x(n) = -a^n u(-n-1)$, then $X(z) = \frac{z}{z-a}$, ROC $= |z| < |a|$.
5. Properties of z-transform: We have $x(n) \overset{Z}{\leftrightarrow} X(z)$ and $ROC_X = r_2 < |z| < r_1$
   - Linearity: $a_1 x_1(n) + a_2 x_2(n) \overset{Z}{\leftrightarrow} a_1 X_1(z) + a_2 X_2(z)$, $ROC \geq ROC_{X_1} \cap ROC_{X_2}$
   - Time shifting: $x(n-k) \overset{Z}{\leftrightarrow} z^{-k} X(z)$, $ROC = ROC_X$ except $z = 0$ or $z = \infty$.
   - Scaling in the z-domain: $a^n x(n) \overset{Z}{\leftrightarrow} X(a^{-1}z)$, $ROC = |a|r_2 < |z| < |a|r_1$
   - Time reversal: $x(-n) \overset{Z}{\leftrightarrow} X(z^{-1})$, $ROC = \frac{1}{r_1} < |z| < \frac{1}{r_2}$

- Differentiation in the z-domain: $nx(n) \overset{Z}{\leftrightarrow} -z\frac{dX(z)}{dz}$, $ROC = ROC_X$
- Convolution: $x_1(n) * x_2(n) \overset{Z}{\leftrightarrow} X_1(z)X_2(z)$, $ROC \geq ROC_{X_1} \cap ROC_{X_2}$
- Correlation: $x_1(n) * x_2(-n) \overset{Z}{\leftrightarrow} X_1(z)X_2(z^{-1})$, $ROC \geq ROC_{X_1(z)} \cap ROC_{X_2(z^{-1})}$

6. 1-sided z-transforms: For a given DT signal $x(n)$
   - $X^+(z) := \sum_{n=0}^{\infty} x(n)z^{-n}$, where z is complex valued.
   - Unique for causal signals
   - One sided z-transform of $x(n)$ is identical to the 2-sided z-transform of $x(n)u(n)$
   - For causal signals, $X^+(z)$ and $X(z)$ are the same
   - ROC of $X^+(z)$ is always exterior of a circle
7. Properties of 1-sided z-transform: We have $x(n) \overset{Z^+}{\leftrightarrow} X^+(z)$
   - Almost all properties of the two-sided z-transform carry over to the one-sided z-transform
   - Time delay: $x(n-k) \overset{Z^+}{\leftrightarrow} z^{-k}[X^+(z) + \sum_{n=1}^{k} x(-n)z^n]$
   - Time advance: $x(n+k) \overset{Z^+}{\leftrightarrow} z^k[X^+(z) - \sum_{n=0}^{k-1} x(n)z^{-n}]$
8. Useful theorems on z-transforms
   - Initial value theorem: If $x(n)$ is causal, then $x(0) = lim_{z\to\infty} X(z)$
   - Final value theorem: $\lim_{n\to\infty} x(n) = \lim_{z\to 1}(z-1)X^+(z)$, if the ROC contains the unit circle
9. Inverse z-transforms: Partial fraction expansion
10. Zero state response vs Zero input response

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