Recitation 9 - EECS 451, Winter 2010

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OUTLINE

- Review of Digital filter design
- Practice problems

Concepts: FIR filter design

1) Windowing

$$h(n) = h_{IDEAL}(n)w(n)$$

for some data window $w(n)$.

2) Frequency sampling

Solve

$$\sum_{-N/2}^{N/2} h(n)e^{-j\omega_k n} = H_D(e^{j\omega_k})$$

for $\omega_k = \frac{2\pi k}{N+1}$.

Concepts: IIR filter design

1) Impulse invariance

$$h(n) = Th_a(nT).$$

2) Bilinear transformation

a)

$$H(z) = H_a\left(s = \frac{2}{T}\frac{z-1}{z+1}\right).$$

b) Frequency warping

$$\Omega = \frac{2}{T}tan(\frac{\omega}{2}).$$

Problems

1) Determine the impulse response $\{h(n)\}$ of a linear-phase FIR filter of length $M = 4$ for which the frequency response at $\omega = 0$ and $\omega = \pi/2$ is specified as

$$H_r(\omega = 0) = 1, \qquad H_r(\omega = \pi/2) = -\frac{\sqrt{2}}{4}(i + j).$$

2) Use the bilinear transformation to convert the analog filter with system function

$$H(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}$$

into a digital IIR filter. Select $T = 0.1$.

3) Repeat 2) using the impulse variance.

The solutions for Recitation 9 by Jung Hyun Bae

1. $H(e^{j\omega_k}) = \sum_{n=0}^{3} h(n) e^{-j\omega_k n}$

$$H(e^{j0}) = \sum_{n=0}^{3} h(n) = 1$$

$$H(e^{j\frac{\pi}{2}}) = \sum_{n=0}^{3} h(n) e^{-j\frac{\pi}{2}n}$$

$$= h(0) - jh(1) - h(2) + jh(3) = -\frac{\sqrt{2}}{4}(1+j)$$

Since the filter is linear phase,

$$h(0) = \pm h(3), \quad h(1) = \pm h(2)$$

i) $h(0) = h(3)$, $h(1) = h(2)$

$$h(0) + h(1) = \frac{1}{2}$$

$$(1+j)h(0) - (1+j)h(1) = -\frac{\sqrt{2}}{4}(1+j)$$

$$h(0) - h(1) = -\frac{\sqrt{2}}{4}$$

$$h(0) = \frac{2-\sqrt{2}}{8} = h(3), \quad h(1) = \frac{2+\sqrt{2}}{8} = h(2)$$

ii) $h(0) = -h(3)$, $h(1) = -h(2)$

$$\sum_{n=0}^{3} h(n) = 0 \neq 1$$

$\therefore$ Impossible.

2. $H(z) = H_a\left(s = 20\,\frac{z-1}{z+1}\right)$

The analog filter has poles at

$$s = -0.1 \pm 3j.$$

Hence, the digital filter has poles at

$$20\,\frac{z-1}{z+1} = -0.1 \pm 3j$$

In other words, poles are

$$z = \frac{19.9+3j}{20.1-3j}, \quad \frac{19.9-3j}{20.1+3j}.$$

The analog filter has a zero at

$$s = -0.1.$$

Hence, the digital filter has a zero at

$$20\,\frac{z-1}{z+1} = -0.1.$$

A zero is

$$z = \frac{19.9}{20.1}$$

$$\therefore\ H(z) = \frac{z - \frac{19.9}{20.1}}{\left(z - \frac{19.9+3j}{20.1-3j}\right)\left(z - \frac{19.9-3j}{20.1+3j}\right)}$$

3. If $$H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k}, \text{ then}$$

$$h_a(t) = \sum_{k=1}^{N} c_k e^{p_k t} \qquad t \geq 0$$

Since $h(n) = T h_a(nT)$,

$$H(z) = \sum_{n=0}^{\infty} T \sum_{k=1}^{N} c_k e^{p_k T n} z^{-n}$$

$$= \sum_{k=1}^{N} T c_k \sum_{n=0}^{\infty} (e^{p_k T} z^{-1})^n$$

$$= \sum_{k=1}^{N} \frac{T c_k}{1 - e^{p_k T} z^{-1}}.$$

Hence, the digital filter has poles at

$z = e^{p_k T}$.

$$H_a(s) = \frac{0.5}{s + 0.1 - 3j} + \frac{0.5}{s + 0.1 + 3j}$$

$$H(z) = \frac{0.05}{1 - e^{-0.01 + 0.3j} z^{-1}} + \frac{0.05}{1 - e^{-0.01 - 0.3j} z^{-1}}.$$

$$= \frac{0.1\left(1 - (e^{-0.01} \cos 0.3) z^{-1}\right)}{1 - (2e^{-0.01} \cos 0.3) z^{-1} + e^{-0.02} z^{-2}}$$