OUTLINE

- Review of Digital filter design
- Practice problems

Concepts: FIR filter design

1) Windowing

\[ h(n) = h_{\text{IDEAL}}(n)w(n) \]

for some data window \( w(n) \).

2) Frequency sampling

Solve

\[ \sum_{-N/2}^{N/2} h(n)e^{-j\omega_k n} = H_D(e^{j\omega_k}) \]

for \( \omega_k = \frac{2\pi k}{N+1} \).

Concepts: IIR filter design

1) Impulse invariance

\[ h(n) = Th_a(nT). \]

2) Bilinear transformation

a) \n
\[ H(z) = H_a \left( s = \frac{2}{T} \frac{z - 1}{z + 1} \right). \]

b) Frequency warping

\[ \Omega = \frac{2}{T} \tan \left( \frac{\omega}{2} \right). \]
Problems

1) Determine the impulse response \( \{h(n)\} \) of a linear-phase FIR filter of length \( M = 4 \) for which the frequency response at \( \omega = 0 \) and \( \omega = \pi/2 \) is specified as

\[
H_r(\omega = 0) = 1, \quad H_r(\omega = \pi/2) = -\frac{\sqrt{2}}{4} (i + j).
\]

2) Use the bilinear transformation to convert the analog filter with system function

\[
H(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}
\]

into a digital IIR filter. Select \( T = 0.1 \).

3) Repeat 2) using the impulse variance.
1. \( H(e^{j\omega}) = \sum_{n=0}^{3} h(n) e^{-j\omega n} \)

\[ H(e^{j\omega}) = \sum_{n=0}^{3} h(n) = 1 \]

\[ H(e^{-j\omega}) = \sum_{n=0}^{3} h(n) e^{-j\omega n} = h(0) - jh(1) - h(2) + h(3) = -\frac{\sqrt{2}}{4}(1 + j) \]

Since the filter is linear phase,

\[ h(0) = \pm h(3), \quad h(1) = \pm h(2) \]

i) \( h(0) = h(3), \quad h(1) = h(2) \)

\[ h(0) + h(1) = \frac{1}{2} \]
\[ (1 + j) h(0) - (1 + j) h(1) = -\frac{\sqrt{2}}{4}(1 + j) \]
\[ h(0) - h(1) = -\frac{\sqrt{2}}{4} \]
\[ h(0) = \frac{2 - \sqrt{2}}{8}, \quad h(1) = \frac{2 + \sqrt{2}}{8} \]
\[ = h(3) = h(2) \]

ii) \( h(0) = -h(3), \quad h(1) = -h(2) \)

\[ \sum_{n=0}^{3} h(n) = 0 \neq 1 \]

Impossible.
2. \( H(z) = H_n \left( s = 20 \frac{z-1}{z+1} \right) \)

The analog filter has poles at

\( s = -0.1 \pm 3j \)

Hence, the digital filter has poles at

\( 20 \frac{z-1}{z+1} = -0.1 \pm 3j \)

In other words, poles are

\( z = \frac{19.9 + 3j}{20.1 - 3j}, \frac{19.9 - 3j}{20.1 + 3j} \)

The analog filter has a zero at

\( s = -0.1 \)

Hence, the digital filter has a zero at

\( 20 \frac{z-1}{z+1} = -0.1 \)

A zero is

\( z = \frac{19.9}{20.1} \)

\[ H(z) = \frac{z - \frac{19.9}{20.1}}{(z - \frac{19.9 + 3j}{20.1 - 3j})(z - \frac{19.9 - 3j}{20.1 + 3j})} \]
\[ h(t) = \sum_{k=1}^{N} C_k e^{\frac{\alpha}{T} t}, \quad t \geq 0 \]

Since \( h(n) = T h_n(nT) \),

\[ H(z) = \sum_{k=0}^{N} T C_k \sum_{n=0}^{\infty} (e^{\frac{\alpha}{T} T})^n z^{-n} \]
\[ = \sum_{k=1}^{N} \frac{T C_k}{1 - e^{\frac{\alpha}{T} T} z^{-1}}. \]

Hence, the digital filter has poles at \( z = e^{\frac{\alpha}{T} T} \).

\[ H_a(s) = \frac{0.5}{s + 0.1 - 3s} + \frac{0.5}{s + 0.1 + 3s} \]

\[ H(z) = \frac{0.05}{1 - \left( e^{-0.01+0.3} \right) z^{-1}} + \frac{0.05}{1 - e^{-0.01-0.3}} z^{-1}. \]

\[ = \frac{0.1 \left( 1 - \left( e^{-0.01 \cos 0.3} \right) z^{-1} \right)}{1 - \left( 2 e^{-0.01 \cos 0.3} \right) z^{-1} + e^{-0.02} z^{-2}}. \]