Recitation 8 - EECS 451, Winter 2010

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OUTLINE

- Using data windows
- Practice problems

Concepts: Frequency analysis using data windows

1) Compute unknown ω_i from

$$y[n] = A_1 cos(\omega_1 n + \theta_1) + A_2 cos(\omega_2 n + \theta_2).$$

by using only $\{y[n], 0 \leq n \leq L-1\}$

- Compute

$$Y_k = \sum_{n=0}^{N-1} y[n] e^{j2\pi nk/N}.$$

- Find peaks in $|Y_k|$ at $k = k_1, k_2 < N/2$. Then, the discrete frequencies are: $\omega_i = \frac{2\pi}{N} k_i$.

2) We can think of

$$y[n] = w[n]\Big(A_1 cos(\omega_1 n + \theta_1) + A_2 cos(\omega_2 n + \theta_2)\Big),$$

where $w[n] = 1$, $0 \leq n \leq L-1$.

- $$W(e^{j\omega}) = \frac{sin(\omega L/2)}{sin(\omega/2)} e^{-j\omega(L-1)/2}$$

3) Resolving peaks.

$$|\omega_1 - \omega_2| > \frac{2\pi}{L}$$

to resolve peaks at ω_1 and ω_2, because the first zero-crossing of $W(e^{j\omega})$ occurs at $\omega = \frac{2\pi}{L}$.

Problems

1) Consider a problem of analyzing frequency spectrum of $y[n] = A_1 cos(\omega_1 n+\theta_1)+A_2 cos(\omega_2 n+\theta_2)$ using N-point DFT where the data is available for $0 \leq n \leq L-1$.
   a) Suppose we increased L. Explain the effect of a larger L.
   b) Explain the effect of a larger N.
   c) Explain the effect of using Hamming window.
2) a) A filter is given by the following difference equation.

$$y(n) + y(n-1) + y(n-1) = x(n) - x(n-1) + x(n-2).$$

   Find the frequency component which this filter rejects.

   b) A filter is given by the following difference equation.

$$2y(n) + 3y(n-1) + ay(n-1) = x(n) + x(n-1) + x(n-2).$$

   Find a which makes the phase response be zero for all frequencies.
3) Let $x(n)$ be the real and even signal with period 6. Find the frequency component of the output of the system $y(n) = x(n) - x(n-1) + x(n-3) - x(n-4)$ using DTFS of $x(n)$.

Solutions for the Recitation 8 by Jung Hyun Bae

1 a) Increasing L improves the frequency resolution. In other words, it makes easier to distinguish two frequency components.

b) Increasing N increases the number of frequency domain samples of the signal spectrum. In other words, we have a finer sampling.

c) The advantage of the Hamming window is that the leakage is smaller. The Hamming window, however, has wider main lobe width than the rectangular window. Hence, the Hamming window makes the spectrum smoother.

2. a) $$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2 - z + 1}{z^2 + z + 1}.$$

$$z^2 - z + 1 = (z - e^{jw_0})(z - e^{-jw_0}) = z^2 - 2(\cos w_0) z + 1$$

$$\therefore \cos w_0 = \frac{1}{2}$$

$$w_0 = \frac{\pi}{3}$$

The filter rejects $\frac{\pi}{3}$.

b) $$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1} + z^{-2}}{2 + 3z^{-1} + az^{-2}}$$

$$H(e^{jw}) = \frac{1 + e^{-jw} + e^{-2jw}}{2 + 3e^{-jw} + ae^{-2jw}}$$

$$= \frac{e^{jw} + 1 + e^{-jw}}{2e^{jw} + 3 + ae^{-jw}}$$

$$= \frac{1 + 2\cos w}{3 + 2e^{jw} + ae^{-jw}}$$

If $a = 2$, then $H(e^{jw}) = \frac{1 + 2\cos w}{3 + 2\cos w}$.

$\angle H(e^{jw}) = 0$.

3. $$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-1} + z^{-3} - z^{-4}$$

$$= \frac{z^4 - z^3 + z - 1}{z^4}$$

$$= \frac{(z-1)(z^3+1)}{z^4}$$

$$= \frac{z^6 - 1}{(z^2+z+1)z^4}$$

$H(z)$ rejects $\frac{k}{3}\pi$ $(k = 0, 1, \cdots, 5)$ except for $\frac{2}{3}\pi, \frac{4}{3}\pi$

Since $x(n)$ is real and even. X_k is also real and even.

Let $X_k = \{a_0, a_1, a_2, a_3, a_2, a_1\}$

Then. $x(n) = a_0 + a_1(e^{j\frac{\pi}{3}n} + e^{-j\frac{\pi}{3}n}) + a_2(e^{j\frac{2}{3}\pi n} + e^{-j\frac{2\pi}{3}n})$

$+ a_3 e^{j\pi n}$

$$H(e^{j\frac{2}{3}\pi}) = \frac{(e^{j\frac{2}{3}\pi} - 1)(e^{j2\pi} + 1)}{e^{j\frac{2}{3}\pi}}$$

$$= \frac{(\frac{1}{2} + \frac{\sqrt{3}}{2}i)2}{(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)}$$

$$= 2(\frac{1}{2} - \frac{\sqrt{3}}{2}i) = 2e^{-j\frac{2}{3}\pi}$$

$$H(e^{-j\frac{2}{3}\pi}) = 2e^{j\frac{2}{3}\pi}$$

$$\therefore y(n) = 4a_2 \cos(\frac{2}{3}\pi n - \frac{2}{3}\pi)$$