Recitation 3 -EECS 451, Winter 2010

Jan 27, 2010

OUTLINE

- Review of important concepts (Lecture 3-4)
- Practice problems

Concepts: Discrete Time Systems

1. Classification of DT systems: For $y(n)=\mathcal{T}(x(n))$,

- static (memoryless): y(n) depends only on the present input
- causal: y(n) depends only on the present and past inputs
- time-invariant: $\mathcal{T}(x(n-k)) = y(n-k)$, where k is an integer
- linear: $\mathcal{T}(a_1x_1(n)+a_2x_2(n)) = a_1\mathcal{T}(x_1(n)) + a_2\mathcal{T}(x_2(n))$
- stable (BIBO): every bounded input provides a bounded output

2. Linear Time Invariant (LTI) System: $y(n)=\mathcal{T}(x(n))$

- completely characterized by the impulse response $h(n)=\mathcal{T}(\delta(n))$
- output given by simple convolution operation, $y(n) := h(n)*x(n)=\sum_{k=-\infty}^{\infty} x(n-k)h(k)$

3. Properties of convolution

- Commutative: $x(n) * h(n)= h(n) * x(n)$
- Associative: $(x(n) * h_1(n)) * h_2(n)= x(n) * (h_1(n) * h_2(n))$
- Distributive: $x(n) * (h_1(n)+ h_2(n)) = x(n) * h_1(n)+ x(n) * h_2(n)$

4. Classification of LTI systems by the impulse response

- Causal: The given LTI system is causal if and only if $h(n)=0\ \forall n< 0$.

- Stable (BIBO): The given LTI system is stable if and only if $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

5. Two classes of LTI systems characterized by the impulse response

- Finite Impulse Response (FIR) system: number of non-zero h(n)'s are finite.
- Infinite Impulse Response (IIR) system: number of non-zero h(n)'s are infinite.

6. The system defined by linear constant coefficient difference equation

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

- is LTI and causal
- can be implemented by direct form I or direct form II (more efficient).
- is recursive and the impulse response is IIR if $N \geq 1$.
- is non-recursive and the impulse response is FIR if $N = 0$. $h(n)= \{\underline{b_0}, b_1, \cdots, b_M\}$.

Problems

1. Determine which of the following systems is static, linear, time-invariant, causal, stable

(a) $y(n)= x^2(n + 1)$

(b) $y(n) = \begin{cases} x(-n), & n < 0 \\ x(n), & n \geq 0 \end{cases}$

(c) $y(n)= \sum_{k=-\infty}^{\infty} x(n - k)p(k)$, where $p(n)= \{-10, \cdots, -1, \underline{0}, 1, \cdots, 10\}$

2. Compute the output of the following LTI systems

(a) $x(n)= \{\underline{0}, 0, 1, 1, 1, 1\}$, $h(n)= \{1, \underline{-2}, 3\}$

(b) $x(n)= \{\underline{1}, 1, 2\}$, $h(n)= u(n)$

3. Determine the impulse response of a discrete-time system realized by the structure shown in Fig. 1.

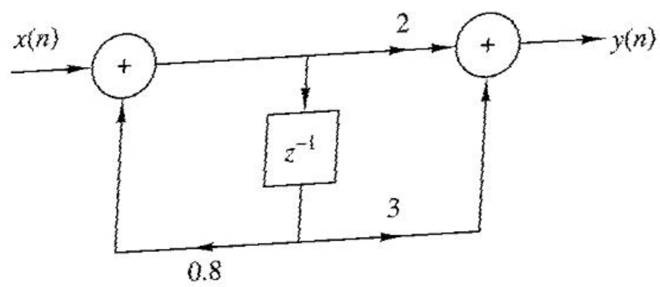


Fig. 1

Solutions for Recitation 3 by Jung Hyun Bae

1. (a) It is not static
($y(n)$ depends on $x(n+1)$)

It is not linear
$\left(2x(n+1) \to \boxed{} \to 4x^2(n+1) \neq 2y(n) = 2x^2(n+1)\right)$

It is time-invariant
$\left(\begin{array}{l} y(n-d) = x^2(n-d+1), \\ \text{let } x_1(n) = x(n-d). \text{ Then } x_1^2(n+1) = x^2(n+1-d) = y(n-d) \end{array}\right)$

It is not causal.
($y(n)$ depends on $x(n+1)$)

It is stable.
(If $x(n)$ is bounded, then $y(n)$ is bounded)

(b) It is not static
($y(n)$ depends on $x(-n)$ for $n<0$)

It is linear.
$\left(a x_1(n) + b x_2(n) \to \boxed{} \to \begin{cases} a x_1(-n) + b x_2(-n), & n<0 \\ a x_1(n) + b x_2(n), & n \geq 0 \end{cases} = a y_1(n) + b y_2(n)\right)$

It is not time-invariant.
$\left(\begin{array}{l} \text{For } n<0,\ d>0, \quad y(n-d) = x(-(n-d)). \\ \text{Let } x_1(n) = x(n-d). \text{ Then } x_1(-n) = x(-n-d) \neq y(n-d) \end{array}\right)$

It is not causal.
($y(n)$ depends on $x(-n)$ for $n<0$)

It is stable.

(c) We can consider this as an LTI system with impulse response $p(n)$.

It is not static
It is linear and time invariant
It is not causal ($p(n) \neq 0$ for $n<0$)
It is Stable. ($p(n)$ is FIR)

2. $y(n) = \sum x(i)\,h(n-i)$

(a) $\{0, \underline{0}, 1, -1, 2, 2, 1, 3\}$

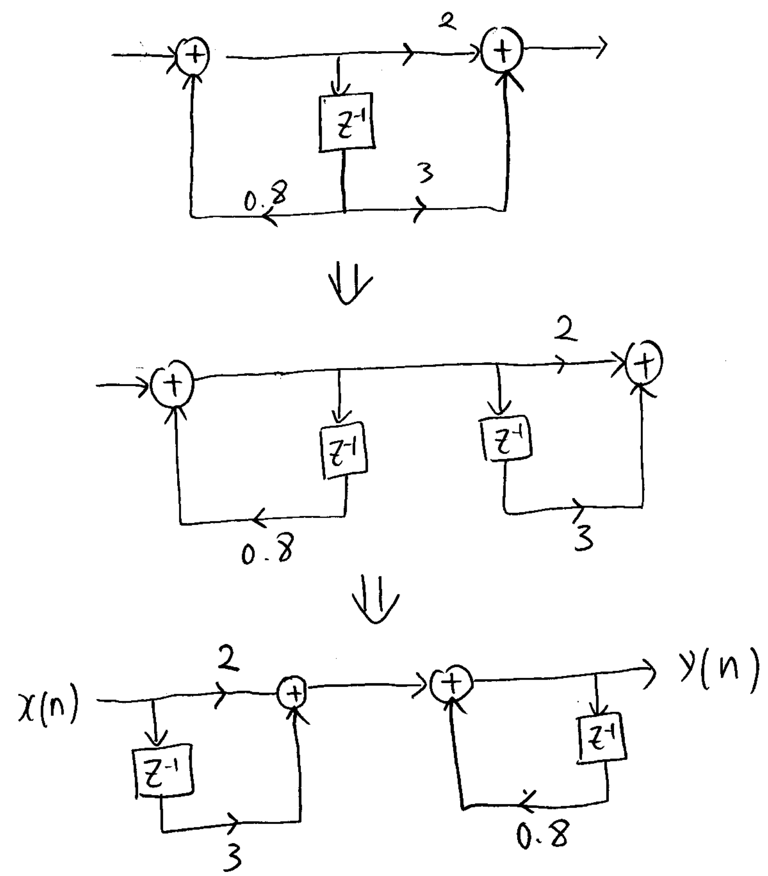
check: $0+0+1-1+2+2+1+3 = 8 = (0+0+1+1+1+1) \times (1-2+3)$

$= 4 \times 2$

(b) $(\delta(n) + \delta(n-1) + 2\delta(n-2)) * u(n)$

$= u(n) + u(n-1) + 2u(n-2)$

3.



$\therefore\ y(n) = 0.8\,y(n-1) + 2x(n) + 3x(n-1)$.

$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$

$h(n) - 0.8h(n-1) = 2\delta(n) + 3\delta(n-1)$

$h(0) = 2$

$h(1) - 0.8h(0) = 3$ $\qquad h(1) = 0.8h(0) + 3$

$h(2) - 0.8h(1) = 0$ $\qquad h(2) = 0.8\,h(1)$

$h(3) = 0.8h(2)$

$\vdots$