Recitation 2 -EECS 451, Winter 2010

Jan 20, 2010

OUTLINE

- Review of Sampling
- Practice problems

Concepts

1. Sampling

- Sampling Theorem
- Ideal Sampling and reconstruction
- Practical issues
- Interpretation of aliasing in both frequency and time domain

Problems

1. A signal $x(t) = \cos(200\pi t)$ is sampled at a rate of ω_s and stored. Is it possible to reconstruct the original signal from the stored samples using an ideal low pass filter when
   (a) $f_s = 85\text{Hz}$
   (b) $f_s = 800\text{Hz}$

2. The following analog sinusoidal signal is sampled 400 times a second and each sample is quantized into 256 different voltage levels

$$x_a(t) = 2\cos(500\pi t) + 3\cos(300\pi t)$$

   (a) Determine the Nyquist sampling rate.
   (b) Determine the folding frequency.
   (c) Determine the frequencies in the resulting discrete-time signal $x(n)$.
   (d) What will be the signal reconstructed assuming an ideal D/A converter and neglecting the quantization effect?

3. A seismic signal with amplitude range [-1, 1] volts is sampled and quantized with a 12 bit A/D converter before transmitting it over a communication channel which can support a maximum bit rate of 240 Kbits/sec. What is the maximum sampling frequency the system can support? What is the maximum frequency that can be present in the original seismic signal in order to avoid any loss of information?

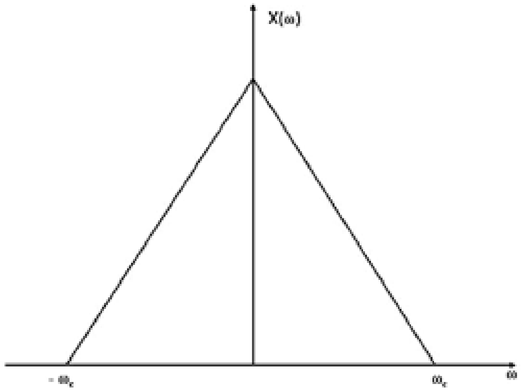
4. A signal x(t) when sampled at a rate $f_s = 1/T_s = \omega_s/2\pi$ produced the samples $\{x[nT_s]\}_{n=-\infty}^{\infty}$.

Let y(t) be a signal constructed from these samples, where

$$y(t) = \frac{\omega_c T_s}{\pi} \sum_{n=-\infty}^{\infty} x[nT_s] \sin c(\frac{\omega_c}{\pi}(t - nT_s))$$

How is y(t) related to x(t) when the frequency spectrum of x(t) is as shown in the figure below with

(i) $\omega_{c=}\ \omega_s/4$, (ii) $\omega_{c=}\ \omega_s$



Solutions for Recitation 2 by Jung Hyun Bae

1. Since the frequency component of $\cos(200\pi t)$ is 100 Hz, the Nyquist rate is 200 Hz.

(a) $85 < 200$. $\therefore$ We cannot reconstruct the original signal

(b) $800 > 200$ $\therefore$ We can " "

2. (a) The frequency component of $\cos(500\pi t)$ is 250Hz, and that of $\cos(300\pi t)$ is 150 Hz.

$\therefore$ The Nyquist rate is 500 Hz.

(b) Since we sample the signal 400 times a second, the sampling rate is 400 Hz and it is not affected by the quantization level.

$\therefore$ The folding frequency = 200 Hz.

(c) $x_a[n] = 2\cos\left(\frac{500\pi}{400}n\right) + 3\cos\left(\frac{300\pi}{400}n\right)$

$= 2\cos\left(\frac{5}{4}\pi n\right) + 3\cos\left(\frac{3}{4}\pi n\right)$

$= 2\cos\left(\frac{3}{4}\pi n\right) + 3\cos\left(\frac{3}{4}\pi n\right) = 5\cos\left(\frac{3}{4}\pi n\right)$ $\therefore$ Frequency $= \frac{3}{8}$ Hz

(d) If we reconstruct the signal using $\frac{1}{400}$ s, then $y(t) = 5\cos(300\pi t)$ which is different from $x_a(t)$.

3. We can sample at $\frac{240000}{12} = 20000$ Hz.

In order to avoid aliasing, the Nyquist rate of the signal should be no larger than 20000 Hz.

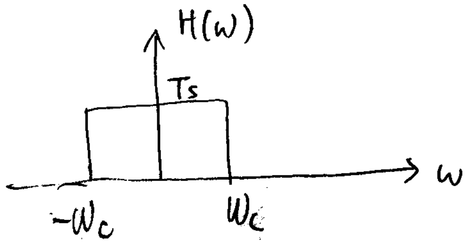
$\therefore$ The possible maximum frequency

$= 10000$ Hz.

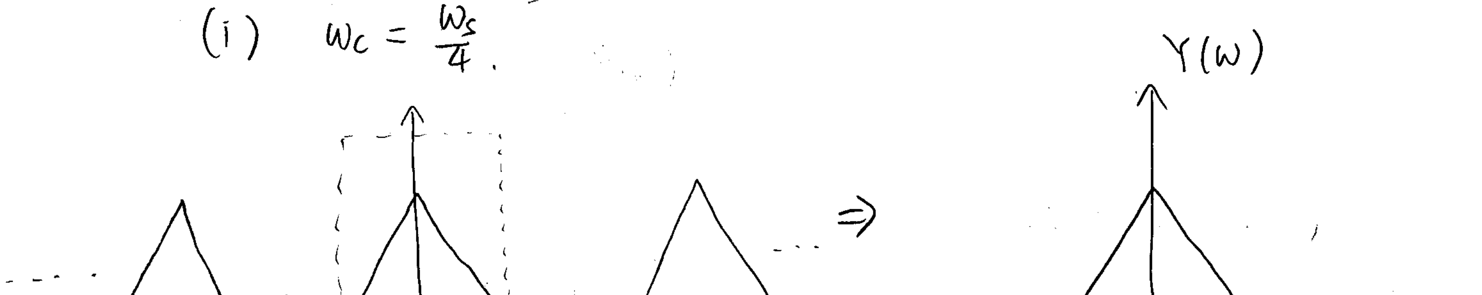
4. $$y(t) = \left[\sum_n x(nT_s)\,\delta(t - nT_s)\right] * \left[\frac{\sin(2\pi f_c t)}{\pi t} T_s\right].$$

Hence, $$Y(\omega) = \frac{1}{T_s}\left[\sum_n X(\omega - \omega_s n)\right] H(\omega)$$

where



(i) $\omega_c = \frac{\omega_s}{4}$.



$\therefore y(t) = x(t)$

ii) $\omega_c = \omega_s$.