Recitation 10 - EECS 451, Winter 2010

Apr. 7, 2010

OUTLINE

- Review of Multirate filter
- Practice problems

Concepts: Downsampling

1) Toss out every Nth sample $\rightarrow$ Increase frequency.

$$y(n) = x(nN).$$

2) In frequency domain, draw $X(e^{j\omega/N})$ (with period $2\pi N$) and add $N-1$ copies.

Concepts: Upsampling

1) Insert $N-1$ zeros between samples.

$$y(n) = \begin{cases} x(n/N) & \text{for } n = kN \\ 0 & \text{otherwise} \end{cases}.$$

2) In frequency domain,

$$Y(e^{j\omega}) = X(e^{j\omega N}).$$

Period becomes $2\pi/N$.

3) We may interpolate the signal after upsampling by applying lowpass filter.

Period becomes 2π.

Concepts: Multirate filtering

1) Change sampling rate by a rational number M/N.
2) Upsample by M; lowpass filter; Downsample by N.
3) The order of upsample and downsample is important (avoid aliasing).

Problems

1) Consider a signal $x(n)$ with its Fourier transform $X(e^{j\omega})$ as in Fig. 1.

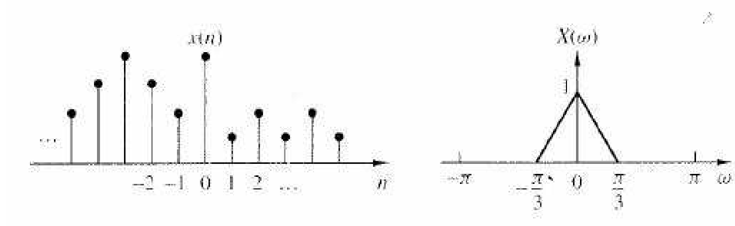


Fig. 1.

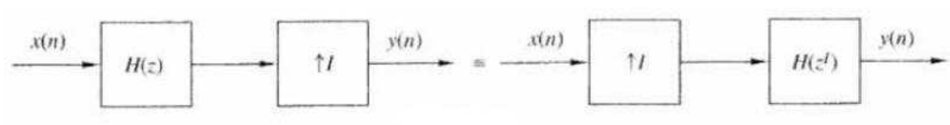


Fig. 2.

a) Sampling $x(n)$ with a sampling period $D = 2$ results in the signal

$$x_s(n) = \begin{cases} x(n) & \text{for } n = 2k \\ 0 & \text{otherwise }. \end{cases}$$

Compute $X_s(e^{j\omega})$. Can we reconstruct $x(n)$ from $x_s(n)$? How?

b) Decimating $x(n)$ by a factor of $D = 2$ produces the signal

$$x_d(n) = x(2n), \text{ all } n.$$

Show that $X_d(e^{j\omega}) = X_s(e^{j\omega/2})$. Do we lose any information when we decimate the sampled signal $x_s(n)$?

2) Prove the equivalence of the two interpolator configurations shown in Fig 2.

3) Consider the two different ways of cascading a downsampler with an upsampler shown in Fig 3. When $D = I$, show that the outputs of the two configuraions are not always the same. Assume an anti-aliasing filter before the downsampler and an interpolating filter after the upsampler.

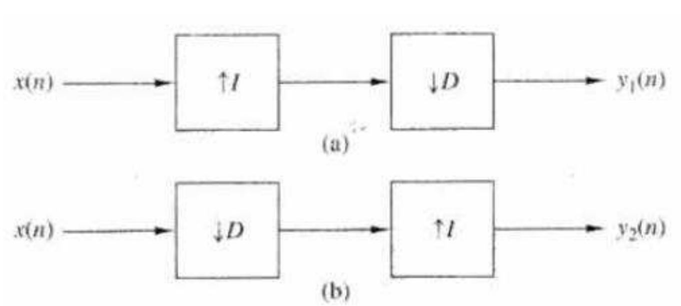


Fig. 3.

Solutions for Recitation 10 by Jung Hyun Bae

1. (a)

$$X_s(e^{jw}) = \sum X_s(n) e^{-jwn}$$

$$= \sum_{n=even} x(n) e^{-jwn}$$

$$= \frac{1}{2}\left(\sum_n x(n) e^{-jwn} + \sum_n (-1)^n x(n) e^{-jwn} \right)$$

$$= \frac{1}{2} \sum_n x(n) e^{-jwn} + \frac{1}{2} \sum_n (e^{j\pi})^n x(n) e^{-jwn}$$

$$= \frac{1}{2} X(e^{jw}) + \frac{1}{2} X(e^{j(w-\pi)})$$

We can reconstrut $x(n)$ from $x_s(n)$ by applying the low pass filter.

(b)

$$X_d(e^{jw}) = \sum X_d(n) e^{-jwn}$$

$$= \sum x(2n) e^{-jwn}$$

$$(\text{Let } n' = 2n)$$

$$= \sum_{n'=even} x(n') e^{-jw\frac{n'}{2}}$$

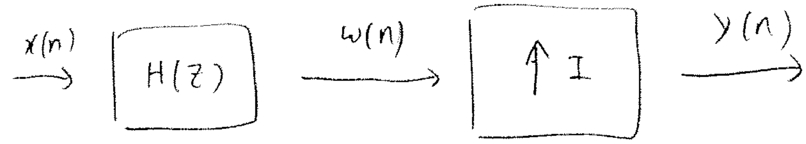
$$= \frac{1}{2}\left(\sum_{n'} x(n') e^{-jw\frac{n'}{2}} + \sum_{n'} (-1)^{n'} x(n') e^{-jw\frac{n'}{2}} \right)$$

$$= \frac{1}{2} X(e^{j\frac{w}{2}}) + \frac{1}{2} X(e^{j(\frac{w}{2}-\pi)})$$

$$= X_s(e^{j\frac{w}{2}})$$

If we decimate $X_s(n)$ instead of $X(n)$, then aliasing happens because the period of $X_s(e^{jw})$ is π.

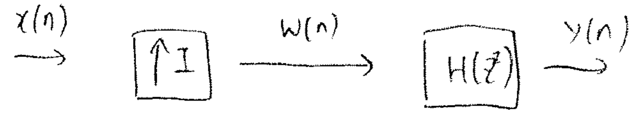
2. i)



$$W(z) = H(z)\,X(z)$$

$$Y(z) = W(z^I) = H(z^I)\,X(z^I)$$

ii)

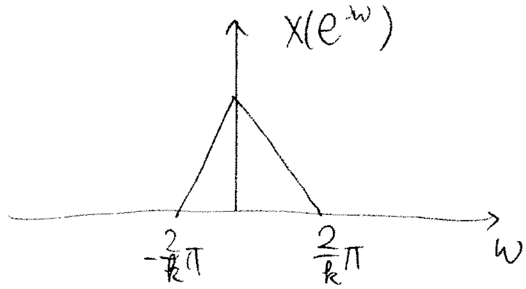


$$W(z) = X(z^I)$$

$$Y(z) = H(z^I)\,W(z)$$

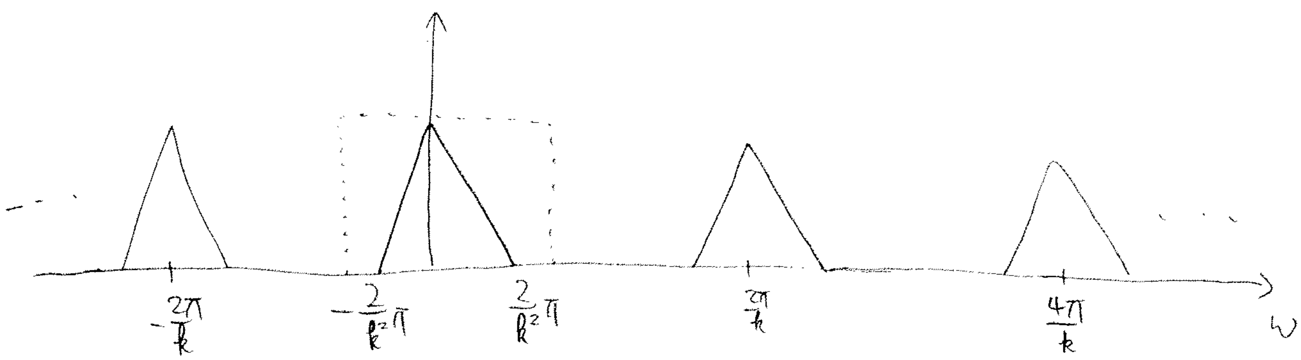
$$= H(z^I)\,X(z^I)$$

3. Let $D = I = k \geq 2$ Consider a signal $x(n)$ whose FT $X(e^{jw})$ is as shown below.

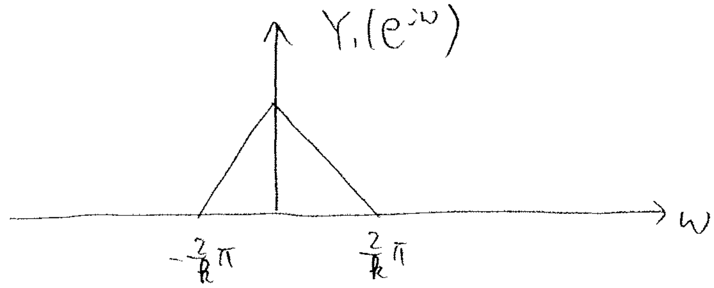


Then, by (a) we get.

After $\boxed{\uparrow I}$

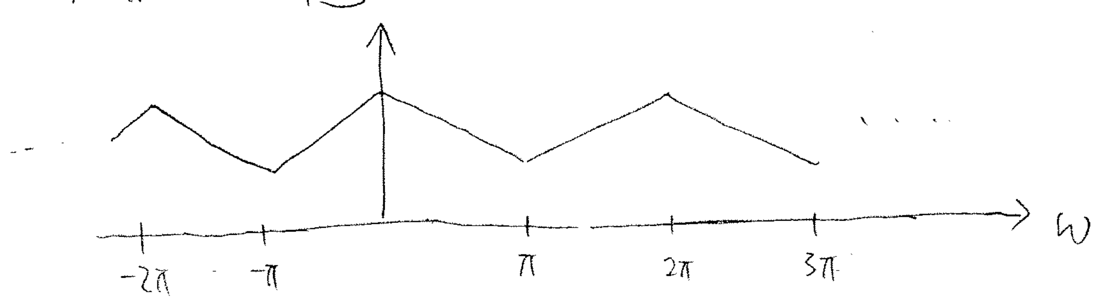


After $\boxed{\downarrow D}$

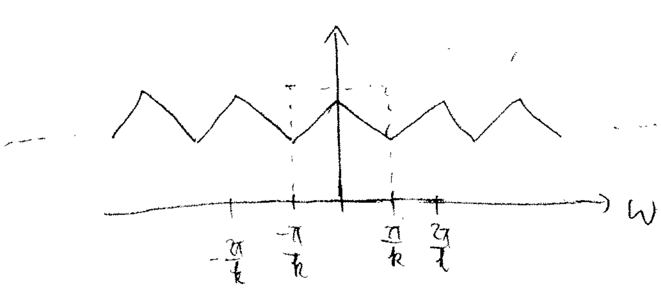


By (b) we get

After $\boxed{\downarrow D}$



After $\boxed{\uparrow I}$



$\Rightarrow$