Topics for Today’s Lecture

- **Spectrogram**: Compute time-varying spectra
- Short-Time-Fourier-Transform (STFT): Spectrogram=|STFT|^2; almost always use
- **Data windows**: Applied to spectrogram

Time-varying spectral content

- “$\text{fit}$” is useful for analyzing periodic signals:
  \[ x(t) = c_0 + c_1 \cos(2\pi T + \theta_1) + c_2 \cos(4\pi T + \theta_2) + \ldots \]
  \[ >> 2/N * \text{abs} (\text{fft}(X,N)) \text{ computes} \ [2c_0, c_1, c_2, \ldots]. \]
- **BUT**: Many (e.g., music) signals look like:
  \[ c_{11} \cos(2\pi T_1 + \theta_{11}) + c_{21} \cos(4\pi T_1 + \theta_{21}) + \ldots \]
  \[ c_{12} \cos(2\pi T_1 + \theta_{12}) + c_{22} \cos(4\pi T_1 + \theta_{22}) + \ldots \]
  \[ c_{13} \cos(2\pi T_1 + \theta_{13}) + c_{23} \cos(4\pi T_1 + \theta_{23}) + \ldots \]

IDEA: Segment (chop up) signal in time.
THEN: Apply “$\text{fit}$” to each time segment.
HOW: \( \text{imagesc} (\text{abs} (\text{fft}(\text{reshape}(X',N,L),N))) \)
WHERE: \( L = \# \text{segments} \) and \( N = \text{length}(X)/L; \)
WHAT: Computes “$\text{fit}$” of each of \( L \) segments.
SHOW: Display freq. vertical, time horizontal.

Example: “The Victors” spectrum

\[
\begin{array}{c}
  \text{Example: “The Victors” spectrogram} \\
  \text{X=Tonal “The Victors” sampled 8192 Hertz.} \\
  \text{26 notes of length 3000/8192 seconds each.} \\
  \text{Length(X)=78000=26(3000). L=26. N=3000.} \\
  \text{>>imagesc(abs(fft(reshape(X',3000,26))))\, ,} \\
  \text{colormap(gray) is shown on the next slide.} \\
  \text{This is called the “spectrogram” of X.}
\end{array}
\]
Example: “The Victors: spectrogram

- Frequency actually displayed from top down, but due to "fft" also displayed from down up.
- Time displayed increasing from left to right.
- Vertical slices are spectra at different times.
- Horizontal slices are presence/absence of a specific frequency as time varies.
- Brightness indicates strength at that time-freq.

Example: chirp signal

- **Chirp**: \( x(t) = \cos(2\pi F t^2) \): birds, dolphins.
- **Frequency** increases linearly with time.
- **Instantaneous frequency** = \(2Ft\) (not \(Ft\)) Hertz.

- \( >> X = \cos([0:8191].^2/10000); \text{plot}(X(1:1000)) \)
- \( >> \text{imagesc}(\text{abs}(\text{fft}(\text{reshape}(X',256,32)))) \)
- \( >> \text{colormap}(\text{gray}) \) shown on next 2 slides.

Chirp signal: time waveform

Chirp signal: spectrogram

**Chirp**: Instantaneous frequency

- \( >> X = \cos([0:8191].^2/10000) \) means this:
- \( x(t) = \cos(t^2) \) sampled at \( t=n/100 \); duration=81.9
- Instantaneous frequency \((2t)/(2\pi)\) Hertz.
- Increases from 0 to \((2(81.9))/(2\pi))=26.08\) Hertz.
- Interpret spectrogram: \( F=100; N=256; T=2.56 \)
- Freq. in final window: \((67-1)100/256=25.8\) Hz. This is average of freqs in final time window.
Vertical slice of final (32nd) time window

Time-Frequency Resolution Tradeoff

- Previous spectrogram used 32 slices @256.
- Next slide spectrogram uses 64 slices @128.
- More resolution in time; less in frequency.
- Note vertical “smearing” in frequency bands.
- **Tradeoff** time vs. frequency resolution by altering #windows=L and N=length(X)/L.
- Leads to next topic of lecture: data windows.

Chirp signal: spectrogram

Windows: Interpretation of data

- **Data windows**: used to aid interpretation of spectra computed from real-world data.
- Deliberately distort the spectrum to aid in interpreting it—determine peak locations.
- The correct answer is not always the best one: What are you attempting to determine?
- What is the problem? See following spectra:

Spectrum: \( \cos(2\pi 440t) \): length=1 sampled at 8192 samples/second

\[
\text{Y} = \cos(2\pi 440\cdot [0:8191]/8192); \text{F} = \text{abs}(\text{fft}(	ext{Y}/8192)); \text{plot}(	ext{F}(1:4096))
\]

Spectrum: \( \cos(2\pi 440.5t) \): length=1 sampled at 8192 samples/second

\[
\text{Y} = \cos(2\pi 440.5\cdot [0:8191]/8192); \text{F} = \text{abs}(\text{fft}(	ext{Y}/8192)); \text{plot}(	ext{F}(1:4096))
\]
Why broadening at the base?

- Do not have an integral number of periods.
- So periodic extension of sinusoid is NOT the same as the sinusoid—discontinuities!
- A sinusoid is only a sinusoid over \(-\infty < t < \infty\)!
- A truncated sinusoid is not same as sinusoid, and it does not have a single-line spectrum.

What to do: Data windows

- **Window** data segment to zero at each end.
- Then no discontinuities in periodic extension.
- Effect of windowing: no base, peak broader.
- Easier to interpret peaks without their bases.
- Use \(N = \text{length}(X); W = \sin(\pi[0:N-1]/(N-1));\)
- Many others: (hanning, hamming, kaiser…)

Spectrum: \(\cos(2\pi 440.5t): \text{length}=1\) sampled at 8192 samples/second

```matlab
>> Y=cos(2*pi*440.5*[0:8191]/8192);Y.*sin(pi*[0:8191]/8192);
>> F=abs(fft(Y/8192)); plot(F(1:4096))
```

Use data window on spectrogram

- Previous high-time low-frequency resolution: \(X = \cos([0:8191]^2/10000); \text{colormap(gray)};\)
- `imagesc(abs(fft(reshape(X',128,64))))`
- Now use data window on each vertical slice:
- \(W = \text{ones(64,1)*sin(pi*[0:127]/127));}\)
- `imagesc(abs(fft(reshape(X',128,64).*W')))`
- Shown on next slide. Note less vertical smearing

Use data window on spectrogram