Given: 2^{nd}-order Butterworth filter: $H_a(s)=\frac{1}{s^2+\sqrt{2}s+1}$.

Goal: Use **bilinear transform** to design a digital lowpass filter with cutoff frequency $\omega_o=\frac{\pi}{2}$.

Sol'n: $1=\Omega_o=\frac{2}{T}\tan(\frac{\omega_o}{2})=\frac{2}{T}\tan(\frac{\pi/2}{2}) \rightarrow T=2.$ $s=\frac{2}{T}\frac{z-1}{z+1}=\frac{z-1}{z+1}$.

Filter: Substitute in $H_a(s)$: $H(z)=\frac{1}{[\frac{z-1}{z+1}]^2+\sqrt{2}[\frac{z-1}{z+1}]+1}\left[\frac{(z+1)^2}{(z+1)^2}\right]$

Simplify: $H(z)=\frac{(z+1)^2}{(z-1)^2+\sqrt{2}(z-1)(z+1)+(z+1)^2}=\boxed{\frac{(z+1)^2}{(2+\sqrt{2})z^2+(2-\sqrt{2})}}$

Zeros: $\{-1,-1\}$ →rejects high freqs. **Poles:** $\{\pm j\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\}$.

Goal: Use a rectangular **window** to design a 5-point (length=5) digital differentiator.

Sol'n: $h_{\text{ideal}}[n]=\text{DTFT}^{-1}\{j\omega\}=\frac{1}{2\pi}\int_{-\pi}^{\pi}(j\omega)e^{j\omega n}d\omega=\frac{(-1)^n}{n}$.

Window: $w[n]=\{1,1,\underline{1},1,1\}$. Then $h[n]=h_{\text{ideal}}[n]w[n]$: $(h[0]=0)$

Answer: $\boxed{h[n]=\{-\frac{1}{2},1,\underline{0},-1,\frac{1}{2}\}}$. Compare to the next slide.

Filter: $y[n]=(x[n+1]-x[n-1])-\frac{1}{2}(x[n+2]-x[n-2])$. Differences!

Goal: Use **frequency sampling** at $\omega=\frac{\pi}{3},\frac{2\pi}{3}$ to design a 5-point digital differentiator.

Sol'n: $H(e^{j\omega})=j\omega, |\omega|<\pi$ is imaginary and odd.

Form: $h[n]$ should be odd. Let $h[n]=\{b,a,\underline{0},-a,-b\}$.

$H(e^{j\omega})=$ $be^{j2\omega}+ae^{j\omega}-ae^{-j\omega}-be^{-j2\omega}=j2b\sin(2\omega)+j2a\sin(\omega)$.

$\omega=\frac{\pi}{3}$: $H(e^{j\pi/3})=j\frac{\pi}{3}=j2a\frac{\sqrt{3}}{2}+j2b\frac{\sqrt{3}}{2} \rightarrow a+b=\frac{\pi}{3\sqrt{3}}$.

$\omega=\frac{2\pi}{3}$: $H(e^{j2\pi/3})=j\frac{2\pi}{3}=j2a\frac{\sqrt{3}}{2}-j2b\frac{\sqrt{3}}{2} \rightarrow a-b=\frac{2\pi}{3\sqrt{3}}$.

Solve: $a=\frac{\pi}{2\sqrt{3}}\approx 0.9$. $b=-\frac{\pi}{6\sqrt{3}}\approx -0.3$ (rounding slightly).

Answer: $\boxed{h[n]=\{-.3,.9,\underline{0},-.9,.3\}}$. Compare to previous slide.

Given: $\cos(2\pi\frac{200}{1000}n)\rightarrow\boxed{\downarrow 3}\rightarrow z[n]\rightarrow\boxed{\uparrow 2}\rightarrow y[n]$.

Goal: Plot spectra of $x[n]$, $z[n]$, $y[n]$. Use $f=\frac{1000\omega}{2\pi}$ Hz.

And: Show output has components at 200 Hz and 300 Hz.

Sol'n: See 3 plots on next slide. All spectra periodic at 1000 Hz.

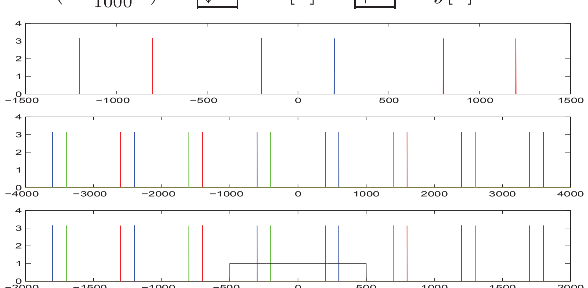
Given: $\cos(2\pi\frac{200}{1000}n)\rightarrow\boxed{\uparrow 2}\rightarrow z[n]\rightarrow\boxed{\text{LPF}}\rightarrow\boxed{\downarrow 3}\rightarrow y[n]$.

Goal: Plot spectra of $x[n]$, $z[n]$, $y[n]$. Use $f=\frac{1000\omega}{2\pi}$ Hz.

And: Show output has components only at 300 Hz.

Sol'n: See 3 plots on next slide. All spectra periodic at 1000 Hz.

Given: $\cos(2\pi\frac{200}{1000}n)\rightarrow\boxed{\downarrow 3}\rightarrow z[n]\rightarrow\boxed{\uparrow 2}\rightarrow y[n]$.



Given: $\cos(2\pi\frac{200}{1000}n)\rightarrow\boxed{\uparrow 2}\rightarrow z[n]\rightarrow\boxed{\text{LPF}}\rightarrow\boxed{\downarrow 3}\rightarrow y[n]$.