Given: $\cos(\frac{\pi}{2}n) \rightarrow \boxed{y[n]+y[n-1]=x[n]-x[n-1]} \rightarrow ?$

Sol'n: $H(e^{j\omega})=\frac{1-e^{-j\omega}}{1+e^{-j\omega}}=\frac{e^{-j\omega/2}}{e^{-j\omega/2}}\frac{e^{+j\omega/2}-e^{-j\omega/2}}{e^{+j\omega/2}+e^{-j\omega/2}}=j\tan(\frac{\omega}{2})$.
$H(e^{j\frac{\pi}{2}})=j\tan(\frac{\pi/2}{2})=j=e^{j\frac{\pi}{2}}$. $\boxed{y[n]=\cos(\frac{\pi}{2}n+\frac{\pi}{2})}$

Design: Notch filter to reject 400 Hz sampled $1200\frac{\text{SAMPLE}}{\text{SECOND}}$.

Sol'n: $-2\cos(2\pi\frac{400}{1200})=1$. Get: $y[n]=x[n]+x[n-1]+x[n-2]$.
Better: $y[n]+0.99y[n-1]+0.99^2y[n-2]=x[n]+x[n-1]+x[n-2]$.

Goal: $DTFT^{-1}[2\cos(\omega)+4\cos(2\omega)+j6\sin(\omega)+j8\sin(2\omega)]$.

Sol'n: Substitute $2\cos(x)=e^{jx}+e^{-jx}$ and $2j\sin(x)=e^{jx}-e^{-jx}$:

Get: $[e^{j\omega}+e^{-j\omega}]+2[e^{j2\omega}+e^{-j2\omega}]+3[e^{j\omega}-e^{-j\omega}]+4[e^{j2\omega}-e^{-j2\omega}]$.

Collect: $(2+4)e^{j2\omega}+(1+3)e^{j\omega}+0+(1-3)e^{-j\omega}+(2-4)e^{-j2\omega}$.

$DTFT^{-1}$: $6e^{j2\omega}+4e^{j\omega}-2e^{-j\omega}-2e^{-j2\omega} \rightarrow \boxed{\{6,4,\underline{0},-2,-2\}}$.

Given: $\frac{1}{4}$-second snippet of signal sampled at $32\frac{\text{SAMPLE}}{\text{SECOND}}$.
Matlab: `fft` output:`[0 0 0 3+4i 0 3-4i 0 0]`. Signal=?

Sol'n: $[32\frac{\text{SAMPLE}}{\text{SECOND}}][\frac{1}{4}\text{SECOND}]=8$ SAMPLES checks.
Freq: `Matlab index` of nonzero in 1^{st} half of output: 4.

Freq: Then $(4-1)\frac{32}{8}=\boxed{12\text{ Hz}}$ Amp. & phase: $3+4i=5e^{j53^o}$.

Amp.: $2\frac{5}{8}=1.25$. **Phase:** 53^o. $\boxed{1.25\cos(2\pi 12t+53^o)}$

Matlab: `fft(1.25*cos(2*pi*12*[0:7]/32+atan(4/3)))`

Given: $\cos(\frac{\pi}{4}n)+\cos(\frac{\pi}{2}n) \rightarrow \boxed{h[n]=\{\underline{a},b,c,d,e\}} \rightarrow 0$.

Sol'n: 2 notch filters in series. Multiply z-transforms.
Use: $-2\cos(\frac{\pi}{4})=-\sqrt{2}$ and $-2\cos(\frac{\pi}{2})=0$ in notches.

Then: $(z^2-\sqrt{2}z+1)(z^2+0z+1)=z^4-\sqrt{2}z^3+2z^2-\sqrt{2}z+1$.

Read off: $\boxed{a=1,b=-\sqrt{2},c=2,d=-\sqrt{2},e=1}$ Symmetric.

OR: Need zeros at $\{e^{\pm j\frac{\pi}{4}},e^{\pm j\frac{\pi}{2}}\}$. Get same answer.

Given: $x[n]=\{\ldots 7,5,2,6,7,5,2,6\ldots\} \rightarrow \boxed{h[n]=\frac{\sin(\frac{\pi}{3}n)}{\pi n}} \rightarrow ?$

Sol'n: $x[n]$ has period $N=4$, so it has the DTFS expansions
DTFS: $x[n]=x_0+x_1e^{j\frac{2\pi}{4}n}+x_2e^{j\frac{4\pi}{4}n}+x_3e^{j\frac{6\pi}{4}n}$. Let $x_1=|x_1|e^{j\theta_1}$.
DTFS: $x[n]=x_0+2|x_1|\cos(\frac{2\pi}{4}n+\theta_1)+x_2\cos(\frac{4\pi}{4}n)$. Note $x_3=x_1^*$.
Since: $x_3e^{j\frac{6\pi}{4}n}=x_1^*e^{-j\frac{2\pi}{4}n}$ and $e^{j\pi n}=\cos(\pi n)$.
Then: sinc lowpass filter eliminates components at $\frac{\pi}{2}$ and π.
So: All that's left is dc term: $y[n]=x_0=\frac{1}{4}(7+5+2+6)=\boxed{5}$

Given: Plot of gain dips to 0 only at $\omega=\{0,\frac{\pi}{2},-\frac{\pi}{2}\}$.
Goal: Give a possible MA system having this gain.

Sol'n: Zeros at $\{0,\frac{\pi}{2},-\frac{\pi}{2}\}$. MA$\rightarrow$no poles (except at origin).
So: $H(z)=(z-1)(z-e^{j\frac{\pi}{2}})(z-e^{-j\frac{\pi}{2}})=(z^3-z^2+z-1)/z^3$.
Note: Put 3 poles at origin $z=0$ to make the system causal.
Answer: $\boxed{y[n]=x[n]-x[n-1]+x[n-2]-x[n-3]}$.