Given: Transfer function $H(z)=\frac{z-1}{z+1}$.
Goal: Poles & zeros, impulse response, difference eqn, $H(e^{j\omega})$.

Sol'n: **Zeros:** $z-1=0 \rightarrow \boxed{\{1\}}$. **Poles:** $z+1=0 \rightarrow \boxed{\{-1\}}$.
h[n]: $H(z)=\frac{z-1}{z+1}=\frac{z+1}{z+1}-\frac{2}{z+1}$. $\mathcal{Z}^{-1}\{H(z)\}=\boxed{\delta[n]-2(-1)^{n-1}u[n-1]}$.
Eqn.: $H(z)=\frac{Y(z)}{X(z)}=\frac{z-1}{z+1}$. Cross-multiply: $Y(z)(z+1)=X(z)(z-1)$.
$\mathcal{Z}^{-1}$: $y[n+1]+y[n]=x[n+1]-x[n] \rightarrow y[n]+y[n-1]=x[n]-x[n-1]$.
$H(e^{j\omega})=\frac{e^{j\omega}-1}{e^{j\omega}+1}=\frac{e^{j\omega/2}}{e^{j\omega/2}}\frac{e^{+j\omega/2}-e^{-j\omega/2}}{e^{+j\omega/2}+e^{-j\omega/2}}=\boxed{j\tan(\frac{\omega}{2})}$.

Goal: Find all inverse z-transforms and ROCs of $X(z)=\frac{z}{z-\frac{1}{3}}+\frac{z}{z-4}$.

Causal: $x[n]=(\frac{1}{3})^n u[n]+(4)^n u[n]$. ROC: $|z|>4$.
Stable: $x[n]=(\frac{1}{3})^n u[n]-(4)^n u[-n-1]$. ROC: $\frac{1}{3}<|z|<4$.
Stable: Since this ROC includes the unit circle $|z|=1$.
Anticausal: $x[n]=-(\frac{1}{3})^n u[-n-1]-(4)^n u[-n-1]$. ROC: $|z|<\frac{1}{3}$.

Given: $\frac{X(z)}{z}=\sum_{i=1}^{N}\frac{A_i}{z-p_i}$ where $|p_1|<|p_2|<\ldots<|p_N|$.
Get: $x_k[n]=\sum_{i=1}^{k}A_i p_i^n u[n]-\sum_{i=k+1}^{N}A_i p_i^n u[-n-1]$ for k=0,1,2...N.
So: There are N+1 possible inverse z-transforms (each value of k).

Goal: Find all inverse systems to $y[n]=x[n]-4x[n-1]$.

Sol'n: $h[n]=\{\underline{1},-4\} \rightarrow H(z)=\frac{z-4}{z}$. Not minimum phase ($|4|>1$).
So: No stable and causal inverse. Can have causal OR stable:
Causal: $G(z)=\frac{1}{H(z)}=\frac{z}{z-4} \rightarrow g[n]=(4)^n u[n] \rightarrow$causal but unstable.
Stable: $g[n]=-(4)^n u[-n-1]=-\{\ldots\frac{1}{1024},\frac{1}{256},\frac{1}{64},\frac{1}{16},\frac{1}{4},\underline{0},0\ldots\}$.

Assume: All values of $|g[n]|<0.001$ are negligible.
Then: $g[n-4]=-\{\underline{\frac{1}{256}},\frac{1}{64},\frac{1}{16},\frac{1}{4}\}$ is causal and MA (length=4).
And: $y[n]*g[n-4]=x[n-4]$ recovers $x[n-4]$ from $y[n]$ (delay=4).

Goal: Compute the causal $Z^{-1}\{\frac{6z^2-14z}{z^2-2z+2}\}$. Let $X(z)=\frac{6z^2-14z}{z^2-2z+2}$.

Poles: $z^2-2z+2=0 \rightarrow z=\{1\pm j\}=\sqrt{2}e^{\pm j\frac{\pi}{4}}$. Let $p=\sqrt{2}e^{j\frac{\pi}{4}}=|p|e^{j\omega}$.
Expand: $\frac{X(z)}{z}=\frac{6z-14}{(z-(1+j))(z-(1-j))}=\frac{A}{z-(1+j)}+\frac{A^*}{z-(1-j)}$.
Residue: $A=\frac{6z-14}{z-(1-j)}|_{z=1+j}=\frac{-8+j6}{j2}=3+j4=5e^{j53^o}=|A|e^{j\theta}$.
Insert: $X(z)=\frac{5e^{j53^o}z}{z-\sqrt{2}e^{j\frac{\pi}{4}}}+\frac{5e^{-j53^o}z}{z-\sqrt{2}e^{-j\frac{\pi}{4}}}$ after multiplying by z.
Answer: $2|A||p|^n\cos(\omega n+\theta)u[n]=\boxed{10(\sqrt{2})^n\cos(\frac{\pi}{4}n+53^o)u[n]}$.

Goal: Compute causal $Z^{-1}\{\frac{6(z-1)}{(z-2)(z-3)}\}$. Let $X(z)=\frac{6(z-1)}{(z-2)(z-3)}$.

#1: $\frac{X(z)}{z}=\frac{6(z-1)}{z(z-2)(z-3)}=\frac{A}{z}+\frac{B}{z-2}+\frac{C}{z-3}$. Divide by z first.
Get: $\frac{X(z)}{z}=\frac{-1}{z}+\frac{-3}{z-2}+\frac{4}{z-3} \rightarrow X(z)=\frac{-1z}{z}+\frac{-3z}{z-2}+\frac{4z}{z-3}$.
Then: $x[n]=-\delta[n]-3(2)^n u[n]+4(3)^n u[n]$. Familiar form.

#2: $X(z)=\frac{6(z-1)}{(z-2)(z-3)}=\frac{D}{z-2}+\frac{E}{z-3}=\frac{-6}{z-2}+\frac{12}{z-3}$.
Get: $x[n]=-6(2)^{n-1}u[n-1]+12(3)^{n-1}u[n-1]$. Same as above!
Note: The impulse makes $x[0]=0$ (initial value theorem).

STABILITY AND CAUSALITY AND ROC

1. Stable$\Leftrightarrow\{|z|=1\}\subset$ ROC.
2. Causal$\Leftrightarrow$ ROC $=\{|z|>A\}$
3. Anticausal$\Leftrightarrow$ ROC $=\{|z|<A\}$
4. $\substack{\text{Causal}\\ \text{Stable}}\Leftrightarrow$ ROC $=\{|z|>A<1\}$
5. $\substack{\text{Causal}\\ \text{Stable}}\rightarrow|p_i|<1$ (poles inside unit circle).
6. $\substack{\text{Anticausal}\\ \text{and Stable}}\rightarrow|p_i|>1$ (poles outside unit circle).
7. Poles inside unit circle$\rightarrow\substack{\text{Causal}\\ \text{Stable}}$ OR $\substack{\text{Anticausal}\\ \text{Unstable}}$.