TOPICS FOR TODAY'S LECTURE

Applications of Multirate:

1. D/A conversion (interpolation)
2. Music signal processing
3. Digital filter design
4. Interpolation in stages

D/A CONVERSION [1/3]

Given: Samples y[n] of $y(t)$
Sampled: Nyquist (minimum) rate
Goal: Continuous-time $y(t)$
Problem: Need *ideal* **analog** lowpass (interpolating) filter!

D/A CONVERSION [2/3]

Idea: Upsample & interpolate y[n]
Then: Only need crude *analog* lowpass filter! (can do).
Trade: Sharp *analog* filter (hard)
for: sharp *digital* filter (easy).

D/A CONVERSION [3/3]

EX: CD "oversampling x4"

x[n]→ [↑ 4]→ [LPF]→ [0–ORDER HOLD INTERPOLATOR]→x(t)

Point: Increases sampling rate to 4(44.1kHz).
Thus: "4x oversampling" (audio term)
→ now can use 0-order hold for D/A.
Also: 3/4 of upsampled x[n] are zeros
→ h[n] convolved with mostly zeros.

AUDIO SIGNAL PROCESSING [1/2]

Given: Snippet of trumpet playing a note.
Goal: Snippets of trumpet playing **all** notes.
Idea: Generate all 12 semitones
using: musical Circle of Fifths
and: Multirate filtering (3/2).
Then: Generate other octaves
using: downsample and upsample.

AUDIO SIGNAL PROCESSING [2/2]

x[n]→[↑ 3]→ [LPF]→ [↓ 2]→y[n]

Does: Reduces frequencies to $\frac{2}{3}$(previous).
Do: This 12 times→12 semitones
via: musical Circle of Fifths.
Then: [↑ 2] and [↓ 2]→different octaves
Used: I used this in my Engin 100 course.

DIGITAL FILTER DESIGN [1/3]

Problem: Sharp lowpass **digital** filters have very long duration h[n].

Needs: Much storage, mults-and-adds.

Also: Output delayed signficantly.

Idea: Use 3 short-duration h[n]

and: downsample in between them.

DIGITAL FILTER DESIGN [3/3]

$H_1(e^{j\omega})$ prevents aliasing after $\boxed{\downarrow \mathbf{10}}$ is really an antialiasing filter.

$H_2(e^{j\omega})$ becomes $H_D(e^{j\omega})$ after $\boxed{\uparrow \mathbf{10}}$ compresses spectrum of z[n].

$H_3(e^{j\omega})$ interpolates following $\boxed{\uparrow \mathbf{10}}$ upsample and interpolate z[n].

INTERPOLATION IN STAGES [2/3]

x[n]→$\boxed{\uparrow \mathbf{5}}$→$\boxed{\mathbf{LPF}}$→$\boxed{\uparrow \mathbf{5}}$→$\boxed{\mathbf{LPF}}$→y[n]

Now: Both LPF cutoff frequencies=$\frac{\pi}{5}$.

and: Dull transitions→short-length h[n].

So: Less storage, fewer mults-and-adds.

Also: Use for antialias and downsampling.

DIGITAL FILTER DESIGN [2/3]

Goal: $H_D(e^{j\omega})=\begin{cases}1 & \text{for } \omega < 0.0095\pi \\ 0 & \text{for } \omega > 0.01\pi\end{cases}$ Compare to $H_2(e^{j\omega})$ below.

Problem: H_D: Sharp→long h[n]. H_2: wider transition→short h[n].

x[n] →$\boxed{H_1(e^{j\omega})}$→ $\boxed{\downarrow \mathbf{10}}$→z[n]→$\boxed{H_2(e^{j\omega})}$→ $\boxed{\uparrow \mathbf{10}}$→$\boxed{H_3(e^{j\omega})}$→y[n]

where: $H_1=\begin{cases}1 & \omega < .0095\pi \\ 0 & \omega > .1\pi\end{cases}$ $H_2=\begin{cases}1 & \omega < .095\pi \\ 0 & \omega > .1\pi\end{cases}$ $H_3=\begin{cases}1 & \omega < .01\pi \\ 0 & \omega > .1\pi\end{cases}$

Why: Sharp LPF w/ large stopband→dull LPF w/ large transition
→ sharp large-stopband LPF after ↑ compresses spectrum.

INTERPOLATION IN STAGES [1/3]

Goal: Upsample & interpolate by 25:

x[n]→$\boxed{\uparrow \mathbf{25}}$→$\boxed{\mathbf{LPF}}$→y[n]

Problem: LPF: Cutoff frequency=$\frac{\pi}{25}$.

Needs: Sharp transition→long h[n].

Idea: Do in two stages:

INTERPOLATION IN STAGES [3/3]

x[n]→$\boxed{\uparrow \mathbf{5}}$→$\boxed{\mathbf{H(z)}}$→$\boxed{\uparrow \mathbf{5}}$→$\boxed{\mathbf{H(z)}}$→y[n]

Same as: x[n]→$\boxed{\uparrow \mathbf{25}}$→$\boxed{\mathbf{HH(z)}}$→y[n]

where: HH(z)=H(z^5)H(z)
→ HH($e^{j\omega}$)=H($e^{j\omega 5}$)H($e^{j\omega}$)

since: $\boxed{\uparrow \mathbf{5}}$ inserts zeros; ×h[n].

Called: Interpolated FIR lowpass filter.