TOPICS FOR TODAY'S LECTURE

Multirate filtering

1. Downsamping
2. Upsampling
3. Multirate
4. Examples

APPLICATIONS

What: Vary sampling rate after the fact

Why? Details next lecture:

1. D/A conversion
2. Digital filter design
3. Audio processing

DOWNSAMPLING [1/3]

What: Downsampling⇔Decimation.

Does: Reduces sampling rate; increases freqs.

How: Toss out all but every N^{th} sample.

EX: $\{3,1,4,1,5,9\} \rightarrow \{3,4,5\}$.

Matlab: `L=length(X);Y=X(1:N:L);`

Formula: y[n]=x[Nn] (y[1]=x[N]; y[2]=x[2N]...)

Same as: Reduce sampling rate by N.

Need: Frequencies$< \frac{1}{2N}$(sampling rate)

DOWNSAMPLING [2/3]

Notation: $x[n] \rightarrow \boxed{\downarrow \mathbf{N}} \rightarrow y[n]$

Does: x[n]=$\cos(\omega_o n)$

$\rightarrow$ y[n]=x[Nn]=$\cos((N\omega_o)n)$.

N=2: $Y(e^{j\omega})=\sum x[2n]e^{-j\omega(2n)/2}$

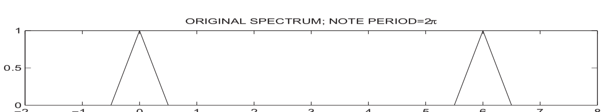
$=\sum x[n']\frac{1}{2}[1+(-1)^{n'}]e^{-j\omega n'/2}$

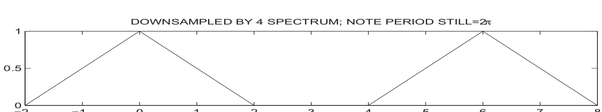
$=\frac{1}{2}X(e^{j\omega/2})+\frac{1}{2}X(e^{j(\frac{\omega}{2}+\pi)})$

Note: $X(e^{j\omega/N})$ has period=$2\pi N$.

Add N–1 copies→period=2π.

DOWNSAMPLING [3/3]





UPSAMPLING [1/3]

What: Upsampling⇔zero-stuffing.

How: Inserts N-1 zeros between samples

Matlab: `YY=[X;zeros(N-1,L)];Y=YY(:);`

EX: $\{3,1,4\} \rightarrow \{3,0,1,0,4,0\}$.

Notation: $x[n] \rightarrow \boxed{\uparrow \mathbf{N}} \rightarrow y[n]$

Formula: y[n]=$\begin{cases} x[n/N] & \text{for n multiple of N} \\ 0 & \text{for otherwise} \end{cases}$

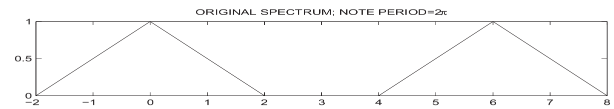
UPSAMPLING [2/3]

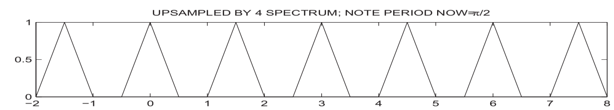
N=2: x[n]=$\cos(\omega_o n)$
$\rightarrow$ y[n]=$\frac{1}{2}[1+(-1)^n]\cos(\omega_o \frac{n}{2})$
$=\frac{1}{2}\cos(\frac{\omega_o}{2}n)+\frac{1}{2}\cos((\pi-\frac{\omega_o}{2})n)$.

N=2: $Y(e^{j\omega}) = \sum_{\text{even}}^{\text{for n}} x[\frac{n}{2}]e^{-j\omega 2(n/2)}$
$=\sum x[n']e^{-j\omega 2n'}=X(e^{j2\omega})\ [n'=\frac{n}{2}]$.

Note: In general, $Y(e^{j\omega})=X(e^{jN\omega})$
is periodic with period=$\frac{2\pi}{N}$.

UPSAMPLING [3/3]





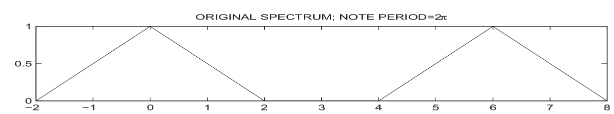
UPSAMPLING & INTERPOLATION [1/2]

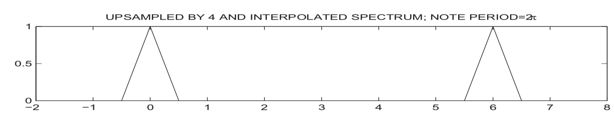
Need: Lowpass filter to remove $\frac{1}{2}\cos((\pi-\frac{\omega_o}{2})n)$.
So: Need to upsample **and interpolate:**

Notation: $x[n] \rightarrow$ [$\uparrow$ **N**] $\rightarrow y[n] \rightarrow$ [**Lowpass filter**] $\rightarrow z[n]$

Matlab:
```
F=fft(Y);Z=real(ifft(F([1:L/N/2+1...
zeros(1,L-L/N-1) L-L/N/2+1:L])));
```

UPSAMPLING & INTERPOLATION [2/2]





MULTIRATE FILTERING [1/2]

1. To change sampling rate
   by a rational number $\frac{M}{N}$:
2. Upsample by M; lowpass filter;
   Downsample by N

Notation: $x[n] \rightarrow$ [$\uparrow$ **M**] $\rightarrow$ [**LPF**] $\rightarrow$ [$\downarrow$ **N**] $\rightarrow y[n]$

Does: Multiplies freqs by $\frac{N}{M}$.

MULTIRATE FILTERING [2/2]

Note: Upsample **first; then** downsample.
Why? Reduce frequencies first,
then increase them (avoids aliasing).
Note: Need to ensure *final* frequency
$< \frac{1}{2}$(sampling rate).
But: Intermediate freqs no problem.

EXAMPLE #1

Given: Sampling rate=2400$\frac{\text{SAMPLE}}{\text{SECOND}}$.

600 Hz $\rightarrow$ **[A/D]** $\rightarrow$ **[$\uparrow$ 3]** $\rightarrow$ **[D/A]** $\rightarrow$?

Soln: $\frac{1}{3}\{600, 2400-600, 2400+600\}$
$=\{200, 600, 1000\}$ Hz.
Want only 200 Hz? Need LPF.

EXAMPLE #2

Given: Sampling rate=2400$\frac{\text{SAMPLE}}{\text{SECOND}}$.

300 Hz $\rightarrow$ **[A/D]** $\rightarrow$ **[$\downarrow$ 3]** $\rightarrow$ **[D/A]** $\rightarrow$?

Soln: 3(300)=900 Hz$< \frac{1}{2}(2400)$
$\rightarrow$ no aliasing. 900 Hz only.

EXAMPLE #3

Given: Sampling rate=2400$\frac{\text{SAMPLE}}{\text{SECOND}}$.

500 Hz $\rightarrow$ **[$\downarrow$ 3]** $\rightarrow$ **[$\uparrow$ 2]** $\rightarrow$ **[LPF]** $\rightarrow$?

Soln: 500$\rightarrow$1500 aliased to 900
$\rightarrow \frac{1}{2}\{900, 2400-900\}=\{450, 750\}$ Hz.

EXAMPLE #4

Given: Sampling rate=2400$\frac{\text{SAMPLE}}{\text{SECOND}}$.

500 Hz $\rightarrow$ **[$\uparrow$ 2]** $\rightarrow$ **[LPF]** $\rightarrow$ **[$\downarrow$ 3]** $\rightarrow$?

Soln: $\frac{1}{2}\{500, 2400-500\}=\{250, 950\}$.
LPF leaves only 250$\rightarrow$750 only.

Note: Upsample+LPF+downsample
in that order for multirate.