TOPICS FOR TODAY'S LECTURE

1. Effects of poles & zeros on frequency response
2. Improved notch filter
3. Improved comb filter
4. Improved lowpass filter

POLES & ZEROS & FREQ. RESPONSE [1/3]

Given: Zeros $\{z_1 \ldots z_M\}$.
Given: Poles $\{p_1 \ldots p_N\}$.
Goal: *Shape* of Gain=$|H(e^{j\omega})|$.

How? "Back of envelope."
Why? Gives insight into effects.

POLES & ZEROS & FREQ. RESPONSE [2/3]

$H(z)$: $H(z)=\frac{(z-z_1)(z-z_2)\ldots(z-z_M)}{(z-p_1)(z-p_2)\ldots(z-p_N)}$.

$H(e^{j\omega})$: $|H(e^{j\omega})|=\frac{|e^{j\omega}-z_1|\ldots|e^{j\omega}-z_M|}{|e^{j\omega}-p_1|\ldots|e^{j\omega}-p_N|}$.

Zeros: $z_m = e^{j\omega_m} \rightarrow |e^{j\omega} - z_m| = 0$ at $\omega = \omega_m$.
Gain: $|H(e^{j\omega_o})|$=0 if there is a zero at $e^{j\omega_o}$.

Poles: p_n=$Ae^{j\omega_n} \rightarrow \frac{1}{|e^{j\omega}-p_n|}=\frac{1}{|A-1|}$ at $\omega = \omega_n$.
Gain: $|H(e^{j\omega_o})|$ large if there is a pole at $Ae^{j\omega_o}$.

POLES & ZEROS & FREQUENCY RESPONSE [3/3]

So: To plot Gain=$|H(e^{j\omega})|$ as a function of ω:

1. Start on unit circle at $\omega = 0 \rightarrow z = e^{j\omega} = 1$ (rightmost).
2. Trace along unit circle counterclockwise (increasing ω)
3. When pass zero $Ae^{j\omega_m}, A \approx 1$, gain dips at ω_m.
4. When pass pole $Ae^{j\omega_n}, A \approx 1$, gain peak at ω_n.
5. TABLE:

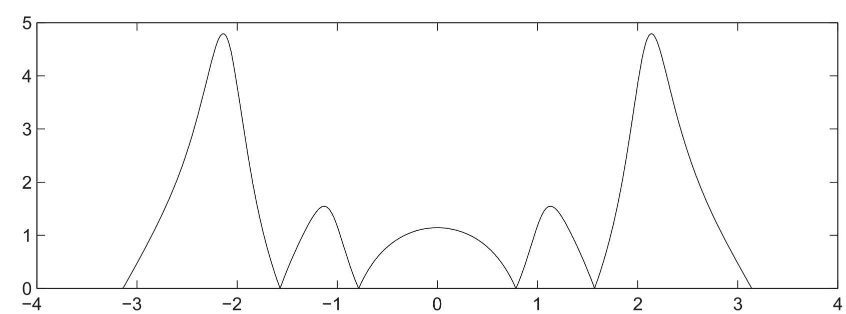
$z = e^{j\omega}$:	$z =$	1	j	-1	$-j$
TABLE:	$\omega =$	0	$\frac{\pi}{2}$	π	$-\frac{\pi}{2}$

EXAMPLE [1/2]

Zeros: $\{e^{\pm j\pi/4}, e^{\pm j\pi/2}, e^{j\pi}\}$.
Poles: $\{0.8e^{\pm j\pi/3}, 0.8e^{\pm j2\pi/3}\}$.

1. Gain$\rightarrow$0 at $\omega = \pm\frac{\pi}{4}; \pm\frac{\pi}{2}$.
2. Gain big at $\omega = \pm\frac{\pi}{3}; \pm\frac{2\pi}{3}$.
3. Closer the zero or pole is to $|z|$=1:
   a. Sharper the peak or dip.
   b. Slower $h[n]$ decays to zero.
4. Gain=$|H(e^{j\omega})|$ on next slide:

EXAMPLE [2/2]: One period of $|H(e^{j\omega})|$:



IMPROVED NOTCH FILTER [1/3]

Given: $x(t) = \sin(2\pi 125t) + \sin(2\pi 200t)$.
Given: DSP system sample at 1000 Hz.
Goal: Keep the 125 Hz; reject 200 Hz.

System: $x(t) \rightarrow \underbrace{\boxed{\text{LPF : 500 Hz}}}_{\text{anti-alias filter}} \rightarrow \underbrace{\boxed{\text{A/D}}}_{\text{sampler}} \rightarrow \underbrace{\boxed{h(n)}}_{\text{filter}} \rightarrow \underbrace{\boxed{\text{D/A}}}_{\text{inter-polate}} \rightarrow y(t)$

IMPROVED NOTCH FILTER [2/3]

Soln: Eliminate 200 Hz$\rightarrow \omega = 2\pi \frac{200}{1000} = 0.4\pi$.

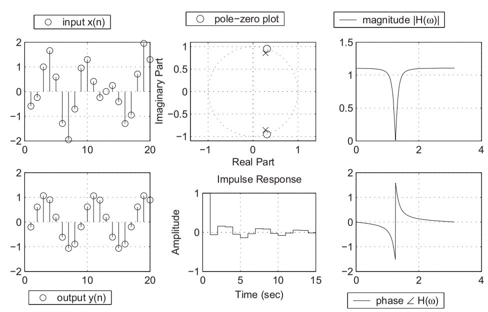
Use: **Zeros:** $e^{\pm j0.4\pi}$. **Poles:** $0.9e^{\pm j0.4\pi}$.

H(z): $\frac{(z-e^{j0.4\pi})(z-e^{-j0.4\pi})}{(z-0.9e^{j0.4\pi})(z-0.9e^{-j0.4\pi})} = \frac{z^2-2z\cos(0.4\pi)+1}{z^2-2z0.9\cos(0.4\pi)+0.81}$.

ARMA: $\mathrm{H(z)} = \frac{Y(z)}{X(z)} = \frac{z^2-0.62z+1}{z^2-0.56z+0.81}$ since $\cos(0.4\pi)=0.31$.

Get: y[n]–0.56y[n–1]+0.81y[n–2]=x[n]–0.62x[n–1]+x[n–2].

IMPROVED NOTCH FILTER [3/3]



LOWPASS FILTER EXAMPLE: [1/4]

Given: DSP system; sample $1000\frac{\text{SAMPLE}}{\text{SECOND}}$.
Given: A chip with 10 storage registers.
Goal: Lowpass filter with cutoff 250 Hz:
Pass-band: $000 < f < 250$ Hertz
Stop-band: $250 < f < 500$ Hertz

Cutoff: Discrete frequency $\omega = 2\pi\frac{250}{1000} = \frac{\pi}{2}$.

LOWPASS FILTER EXAMPLE: [2/4]

Zeros: $\{e^{\pm j\pi/2},\ e^{\pm j3\pi/4},\ e^{j\pi}\}$ stop high frequencies.
Poles: $\{.6, .8e^{\pm j\pi/4}, .8e^{\pm j\pi/2}\}$ pass low frequencies.

H(z): $\frac{(z-e^{j\pi/2})(z-e^{-j\pi/2})(z-e^{j3\pi/4})(z-e^{-j3\pi/4})(z-e^{j\pi})}{(z-.8e^{j\pi/2})(z-.8e^{-j\pi/2})(z-.8e^{j\pi/4})(z-.8e^{-j\pi/4})(z-.6)}$.

Use: $(z-e^{j\pi/2})(z-e^{-j\pi/2})(z-e^{j\pi}) = (z^2+1)(z+1) = z^3+z^2+z+1$.
and: $(z-e^{j3\pi/4})(z-e^{-j3\pi/4}) = z^2-2\cos(\frac{3\pi}{4})z+1 = z^2+\sqrt{2}z+1$.
Also: Performing similar computations for denominator.

LOWPASS FILTER EXAMPLE: [3/4]

H(z): $\frac{z^5+2.414z^4+3.414z^3+3.414z^2+2.414z+1}{z^5-1.73z^4+1.96z^3-1.49z^2+0.84z-0.25} = \frac{Y(z)}{X(z)}$. Cross-multiply:

ARMA: y[n]–1.73y[n–1]+1.96y[n–2]–1.49y[n–3]+0.84y[n–4]–0.25y[n–5]=

Get: x[n]+2.414x[n–1]+3.414x[n–2]+3.414x[n–3]+2.414x[n–4]+x[n–5]

LOWPASS FILTER EXAMPLE: [4/4]

```
Z=[exp(j*pi/2),exp(-j*pi/2),exp(3j*pi/4),exp(-3j*pi/4),exp(j*pi)]
P=[.8*exp(j*pi/2),.8*exp(-j*pi/2),.8*exp(j*pi/4),.8*exp(-j*pi/4),.6]
B=poly(Z);A=poly(P);[H,W]=freqz(B,A);plot(W,abs(H)),zplane(B,A)
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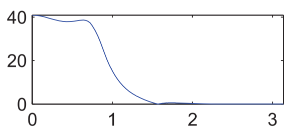

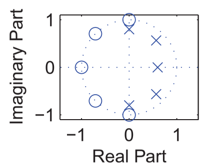

COMB FILTER EXAMPLE: [1/5]

Given: 48 Hz sinusoid+60 Hz interference.
But: Interference periodic: 4 harmonics.

Given: DSP system; sample $480\frac{\text{SAMPLE}}{\text{SECOND}}$.
Given: A chip with 14 storage registers.

Goal: Eliminate periodic interference.

COMB FILTER EXAMPLE: [2/5]

Need: To eliminate frequencies:

1. $\omega = 2\pi\frac{60}{480} = \frac{\pi}{4}$ (fundamental)
2. $\omega = 2\pi\frac{120}{480} = \frac{2\pi}{4}$ (harmonic)
3. $\omega = 2\pi\frac{180}{480} = \frac{3\pi}{4}$ (harmonic)
4. $\omega = 2\pi\frac{240}{480} = \frac{4\pi}{4}$ (harmonic)

Comb: y[n]=x[n]–x[n–8] (eliminate DC) or use
Comb: y[n]=x[n]+x[n–1]+…+x[n–7] (keep DC).

COMB FILTER EXAMPLE: [3/5]

Zeros: $\{e^{\pm j\pi/4}, e^{\pm j\pi/2}, e^{\pm j3\pi/4}, e^{j\pi}\}$.

Poles: $\{.9e^{\pm j\pi/4}, .9e^{\pm j\pi/2}, .9e^{\pm j3\pi/4}, .9e^{j\pi}\}$.

H(z): $\frac{(z-e^{j\pi/4})(z-e^{-j\pi/4})(z-e^{j\pi/2})(z-e^{-j\pi/2})(z-e^{j3\pi/4})(z-e^{-j3\pi/4})(z-e^{j\pi})}{(z-.9e^{j\pi/4})(z-.9e^{-j\pi/4})(z-.9e^{j\pi/2})(z-.9e^{-j\pi/2})(z-.9e^{j3\pi/4})(z-.9e^{-j3\pi/4})(z-.9e^{j}}$

Use: $(z-e^{j\pi/4})\ldots(z-e^{j8\pi/4}) = z^8 - 1$
and: $(z-e^{j\pi/4})\ldots(z-e^{j7\pi/4}) = \frac{z^8-1}{z-1}$
$= z^7+z^6+z^5+z^4+z^3+z^2+z+1.$

COMB FILTER EXAMPLE: [4/5]

H(z): $\frac{z^7+z^6+z^5+z^4+z^3+z^2+z+1}{z^7+.9z^6+.9^2z^5+.9^3z^4+.9^4z^3+.9^5z^2+.9^6z+.9^7}$=H(z)=$\frac{Y(z)}{X(z)}$. Cross-mult.:

ARMA y[n]+(.9)y[n–1]+$(.9)^2$y[n–2]+…+$(.9)^6$y[n–6]+$(.9)^7$y[n–7]
=x[n]+x[n–1]+x[n–2]+…+x[n–5]+x[n–6]+x[n–7]

```
E=exp(j*pi/4);Z=[E,E^2,E^3,E^4,E^5,E^6,E^7]; %7 zeros
P=.9*Z; N=0:79; S=[3,1,4,1,5,-9,2,-7]/2; %0-mean interference
X=1+cos(.2*pi*N)+[S S S S S S S S S S];M=60:79; %repeat period
subplot(421),stem(M,X(M)); B=poly(Z);A=poly(P); %start at 60
subplot(424),zplane(B,A); [H,W]=freqz(B,A); %to allow transient
subplot(422),plot(W,abs(H)); Y=filter(B,A,X); %to decay to 0
subplot(423),stem(M,real(Y(M))); %Interference rejected!
```

COMB FILTER EXAMPLE: [5/5]