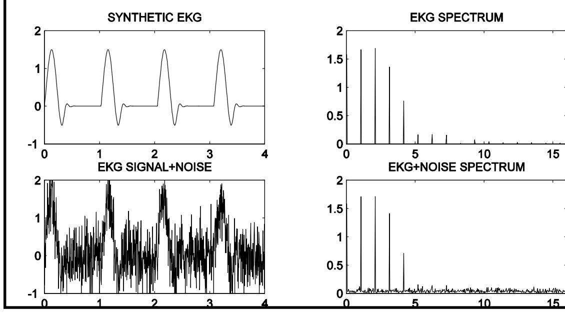
Introduction to EECS 451

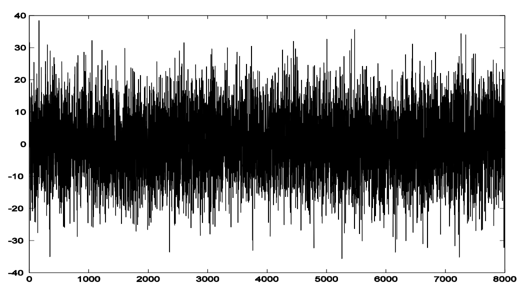
- TOPICS FOR TODAY'S LECTURE:

1. Go over features of the course web site at www.eecs.umich.edu/~aey/eecs451.html Includes: Copies of lecture presentations; Condensed versions of lecture presents; Longer (14 pages) notes on several subjects
2. Introduction to some applications of DSP
3. Periods of discrete-time sinusoids (nontrivial)

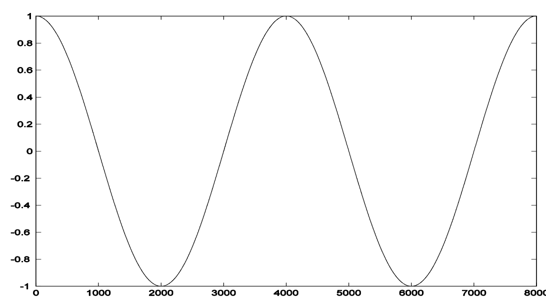
So what can you DO with DSP? 1. You can filter noisy data:



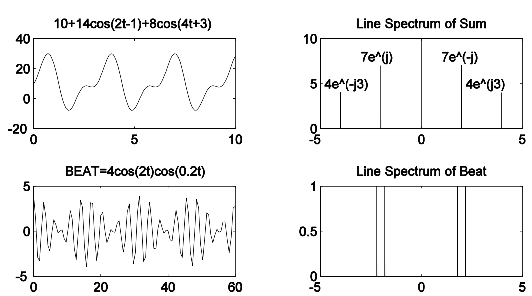
Plotted below is a signal-plus-noise. Can you figure out what the signal is?



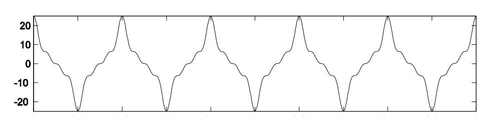
This is the signal without noise.



So what can you DO with DSP? 2. You can compute spectra:

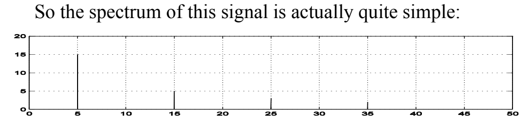


Example: Spectrum of periodic signal



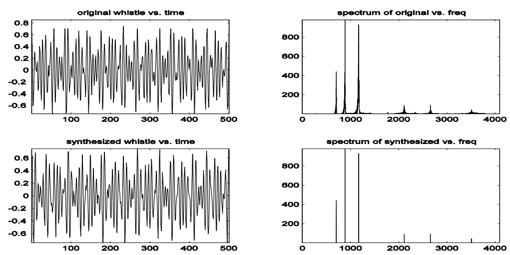
Period=0.2 second (5 complete periods occur in 1 second). Other than that, all you can say is that this looks weird. But:

x(t)=15cos(10πt)+5cos(30πt)+3cos(50πt)+2cos(70πt), 0<t<1. So the spectrum of this signal is actually quite simple:

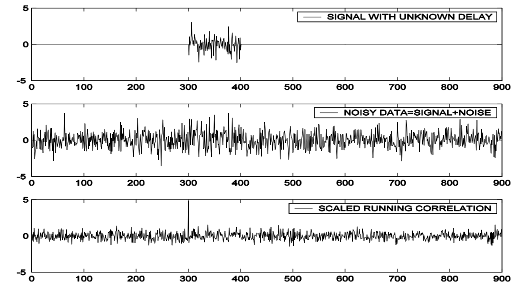


Great! But, given this waveform, how do I compute that formula?

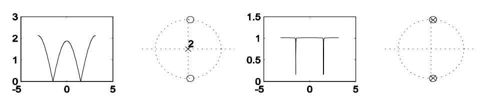
So what can you DO with DSP? 3. You can compress signals:



So what can you DO with DSP? 4. You can detect signals in noise:



So what can you DO with DSP? 5. You can design various filters:



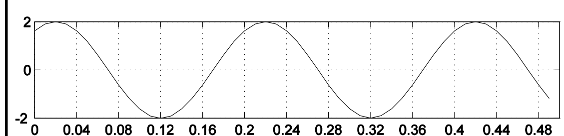
Frequency response of FIR filter: Rejects one frequency.

Pole-zero diagram of FIR filter

Frequency response of IIR filter: Rejects one frequency

Pole-zero diagram of IIR filter

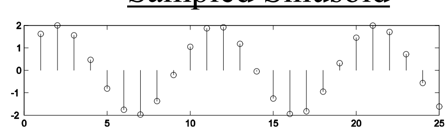
Continuous-time sinusoids



Formulae for this:
$x(t)=2\cos(10\pi t-\pi/5)$
$x(t)=2\cos(10\pi(t-0.02))$

Amplitude=2
Frequency=5 Hertz
Period=0.2 seconds

Sampled Sinusoid



$x(t) = 2\cos(10\pi t-\pi/5)$. Substitute $t=n/50=0.02n$:

$x[n]=2\cos(0.2\pi n-\pi/5)$. Sampling rate=50 Hertz

$x[n]$	$x[0]=1.62$	$x[1]=2.00$	$x[2]=1.62$	$x[3]=0.62$
$x[n]$	$2\cos(0.2\pi 0-\pi/5)$	$2\cos(0.2\pi 1-\pi/5)$	$2\cos(0.2\pi 2-\pi/5)$	$2\cos(0.2\pi 3-\pi/5)$

Periods of sinusoids [1/4]

- $x(t)=2\cos(0.3\pi t+1)$: Period$=2\pi/(0.3\pi)=20/3$.
- $x[n]=2\cos(0.3\pi n+1)$: Period$=2\pi/(0.3\pi)=20/3$.
- *What's wrong with this picture?*
- Period of discrete-time signals must be integer!
- Smallest integer N such that $x[n]=x[n+N]$ is 20.
- $x[n+20]=2\cos(0.3\pi(n+20))=2\cos(0.3\pi n+6\pi)$.

Periods of sinusoids [2/4]

- Period of the discrete-time sinusoid $x[n]=A\cos(\omega n+\theta)$ is computed as follows:

1. Write $2\pi/\omega$=rational number=N/D where N and D are relatively prime. In other words: N/D has been reduced to lowest terms.
2. Then period=N=numerator of fraction N/D.

- If $\omega\neq$(rational number)π, then the sinusoid x[n] NOT periodic! Unlike continuous time.

Periods of sinusoids [3/4]

- Suppose: $A\cos(\omega n+\theta)=A\cos(\omega(n+N)+\theta)$.
- Recall: $\cos(\theta)=\cos(2\pi D+\theta)$ for any *integer* D and for any θ (precludes $\sin(\pi/3)=\sin(2\pi/3)$).
- Need: $(\omega(n+N)+\theta)-(\omega n+\theta)=\omega N=2\pi D$, which becomes $2\pi/\omega=N/D$. Want *smallest* N and D.
- Then: $A\cos(\omega(n+N)+\theta)=A\cos(\omega n+2\pi D+\theta)$.

Periods of sinusoids [4/4]

- EXAMPLES:

1. $x[n]=2\cos(0.3\pi n+1)$: Period=$2\pi/(0.3\pi)=20/3$ already reduced to lowest terms. Period=20.
2. $x[n]=2\cos(0.12\pi n+1)$: Period=$2\pi/(0.12\pi)=$ $100/6=50/3$ after lowest terms. Period=50.
3. $x[n]=2\cos(0.3n+1)$: x[n] is NOT periodic!

Period of Sum of Discrete Sinusoids

- $x[n]=A_1\cos(\omega_1 n+\theta_1)+\ldots+A_N\cos(\omega_N n+\theta_N)$
- For x[n] to be periodic, need the following:
- All ω_i must have forms $2\pi N_i/D_i$ (lowest terms).
- i^{th} term then has period=D_i (before: $2\pi/\omega=N/D$).
- x[n] has period=least common multiple of D_i.
- Least common multiple of set of numbers= (their product)/(their greatest common divisor).