TOPICS FOR TODAY'S LECTURE

Image Processing:

1. Noise filtering, spectrum
2. Deconvolution, filtering
3. Edge detection
4. Median filtering

IMAGE PROCESSING [1/2]

Generalize from 1-D to 2-D:

1. Sampling theorem, Fourier xform.
2. LTI, $\delta[n]$, h[n], convolution.
3. DTFT, DFT, FFT, z-transform.
4. H(z), frequency response, filters.
5. Noise filtering, deconvolution

IMAGE PROCESSING [2/2]

DON'T Generalize usefully:

1. Laplace xform, initial conds.
2. Poles and zeros, partial fracs.
3. ARMA difference equations.
4. Lower half of "Atlanta airport."

Matlab: `A=imread('mandrill.tif','tif');`

IMAGES=2-D SIGNALS

Image=2-D signal z[m,n].
Usually has finite support:
$0 \leq m, n \leq$M–1 ($M \times M$).

Sample continuous-space z(x,y):
z[m,n]=z(x=mT,y=nT); T small.
A/D and D/A same as in 1-D.

2-D CONVOLUTION

Impulse: $\delta[m,n]$=1 if m=n=0.
PSF: Point-Spread Function:

System: δ[m,n]$\rightarrow\boxed{\mathbf{h[m,n]}}\rightarrow$h[m,n].

That is: PSF=2-D impulse response.
Convo-: y[m,n]=h[m,n]**x[m,n] means:
lution: y[m,n]=$\sum_i \sum_j$h[i,j]x[m-i,n-j].

2-D SPECTRUM [1/3]

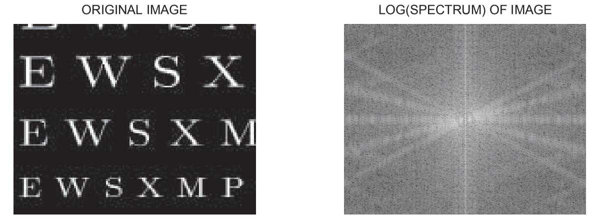
DSFT: $X(e^{j\omega_1}, e^{j\omega_2})=$
$\sum_m \sum_n$x[m,n]$e^{-j(\omega_1 m+\omega_2 n)}$.
DSFT: Discrete Space Fourier Transform.
Note: Periodic in both ω_1 and ω_2.
DFT: $X_{k_1,k_2}=$
$\sum_m \sum_n$x[m,n]$e^{-j\frac{2\pi}{N}(k_1 m+k_2 n)}$.
So: $X_{k_1,k_2}=X(e^{j\omega_1}, e^{j\omega_2})|_{\substack{\omega_1=2\pi k_1/N \\ \omega_2=2\pi k_2/N}}$.

2-D SPECTRUM [2/3]

2-D DFT: `FX=fft2(X,M,N);`
2-D IDFT: `FX=ifft2(X,M,N);`
Center DC: `FY=fftshift(FX);`
Display: `imagesc(X)`, axis off
Gray: `colormap(gray)`
Note: Most images oversampled.

2-D SPECTRUM [3/3]

EX:
```
imagesc(X),colormap(gray),axis off
imagesc(fftshift(log(abs(fft2(X)))))
```



CRUDE 2-D FILTERING [1/2]

EX: $h[m,n]=\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{1} & 1 \\ 1 & 1 & 1 \end{bmatrix}$
Note: **Separable**: $h[m,n]=h[m]h[n]$.
Then: 2-D decouples to 1-D; apply 1-D:
So: $H(e^{j\omega_1}, e^{j\omega_2})=H(e^{j\omega_1})H(e^{j\omega_2})=$ $(1+2\cos(\omega_1))(1+2\cos(\omega_2))$.
Used as: Crude 2-D lowpass filter (notch?).

CRUDE 2-D FILTERING [2/2]

EX: Apply above filter to noisy image:
```
Y=X+600*rand(256,256);Z=conv2(Y,ones(5,5));
```

DFT 2-D FILTERING [1/2]

But: Image proc. seldom real-time.
So: Just set parts of fft2(X) to 0.
Could: Attain brick-wall 2-D LPF!
Don't: Get "ringing" in image.
Do: *Window* spectrum to zero.

DFT 2-D FILTERING [2/2]

EX: Set high wavenumbers to zero:
```
Y=X+600*rand(256,256);FY=fft2(Y);L=20;
FY(L:258-L,L:258-L)=0;Z=real(ifft2(FY));
```

IMAGE DECONVOLUTION [1/3]

Given: y[m,n]=h[m,n]**x[m,n].
From: A medical or optical imaging system of some kind.
where: h[m,n] is known PSF.
since: Measure PSF h[m,n] by imaging a small bead.
Goal: Compute x[m,n] from y[m,n].

IMAGE DECONVOLUTION [2/3]

Soln: If y[m,n] is $N \times N$:
Just: `X=real(ifft2(fft2(Y)./fft2(H,N,N)));`
That is: $X_{k_1,k_2}=Y_{k_1,k_2}/H_{k_1,k_2}$.

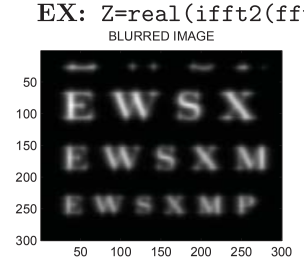
Problem: $H_{k_1,k_2}=0$ for some values of (k_1,k_2)!
Problem: $H_{k_1,k_2} \approx 0 \rightarrow 1/H_{k_1,k_2}$ large.
so: Any noise in Y_{k_1,k_2} will be amplified!

IMAGE DECONVOLUTION [3/3]

Soln: Use $X_{k_1,k_2}=Y_{k_1,k_2}\frac{H^*_{k_1,k_2}}{|H_{k_1,k_2}|^2+\epsilon^2} \approx \frac{Y_{k_1,k_2}}{H_{k_1,k_2}}$.
unless: $H_{k_1,k_2} \approx 0$. Called a **Wiener filter**.
Does: $\mathrm{MIN}(\sum_{k_1,k_2}|Y_{k_1,k_2}-H_{k_1,k_2}X_{k_1,k_2}|^2+\epsilon^2|X_{k_1,k_2}|^2)$.
Called: Tikhonov *regularization*.
Means: Add bias to reduce variance.

IMAGE DECONVOLUTION: EX #1

PSF: `H=.8.^abs(-22:22);H=H'*H;Y=conv2(X,H);`
EX: `Z=real(ifft2(fft2(Y)./fft2(H,300,300)));`



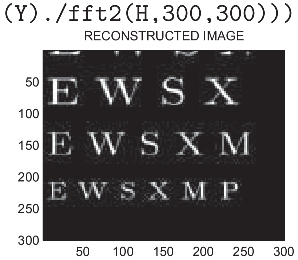
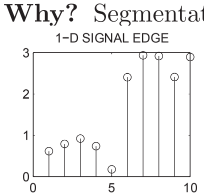


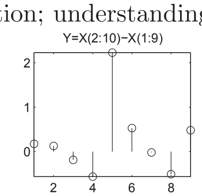
IMAGE DECONVOLUTION: EX #2

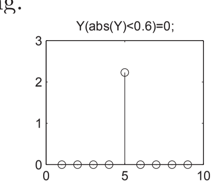
Noise: `Y=Y+600*rand(300,300);FY=fft2(Y);FH=fft2(H,300,300);`
EX: `FZ=FY.*conj(FH)./(abs(FH).^2+10);Z=real(ifft2(FZ));`

EDGE DETECTION: [1/3]

What: Sharp change in values
Why? Segmentation; understanding.







EDGE DETECTION: [2/3]

2-D: Use directional gradients
for: *directional* edge detection:

1. ∇_z[i,j]=(z[i+1,j]–z[i–1,j])
   +(z[i+1,j+1]–z[i–1,j+1])
   +(z[i+1,j–1]–z[i–1,j–1])$\approx \frac{\partial z}{\partial x}$
2. If $|\nabla_z[i,j]| > \eta$, declare $\exists$"vertical edge."
3. Local maxima of $|\nabla_x(i,j)|$: "edge thinning."

Matlab: `edge(A,'sobel',n,'vertical')`

MEDIAN FILTERING [1/2]

For: Impulsive="salt-and-pepper"
=shot noise (arises from bit errors).
while: Preserving image *edges*.
NOTE: Example of *nonlinear* filtering.

1-D: y[n]=median(x[n+2],x[n+1],x[n],x[n-1],x[n-2]).
Why? Preserves edges: u[n] unaffected.
2-D: Apply 1-D to rows, then columns.
Matlab: `medfilt2(A,5,5)`

CIRCULAR SYMMETRY [1/2]

Radial: $H(e^{j\omega_1},e^{j\omega_2})=\begin{cases} 1 & \text{for } \sqrt{\omega_1^2+\omega_2^2} < B \\ 0 & \text{for } \sqrt{\omega_1^2+\omega_2^2} > B \\ & 0 \leq |\omega_1|,|\omega_2| \leq \pi \end{cases}$

Note: $H(e^{j\omega_1},e^{j\omega_2})$="pill-box" (circulary symmetric)

Then: $h[i,j] = \frac{B}{2\pi}\frac{J_1(BR)}{R}$ (circularly symmetric)
where: $R=\sqrt{i^2+j^2}$. (Derivation: Lim p.57).
and: $J_1(\cdot)$=Bessel function of 1^{st} kind of order 1.

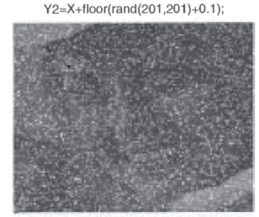
EDGE DETECTION: [3/3]

OR: Find zero-crossings of *Laplacian*:
since: ∇^2x[i,j]$\Leftrightarrow$gradient extrema.

1. ∇^2x[i,j]=x[i+1,j]+x[i–1,j]
   +x[i,j+1]+x[i,j–1]–4x[i,j].
2. Find *zero-crossings* of ∇^2x[i,j].
3. If σ_x^2[i,j]=$\sum_{i-2}^{i+2}\sum_{j-2}^{j+2}$(x[m,n]–$\bar{x}[m,n])^2 > \eta$

then: declare $\exists$"edge" at [i,j].
Matlab: `edge(A,'log')`

Y2=X+floor(rand(201,201)+0.1);



medfilt2(Y2)

CIRCULAR SYMMETRY [2/2]

Note: $H(e^{j\omega_1},e^{j\omega_2})$=
$\begin{cases} H(\sqrt{\omega_1^2+\omega_2^2}) & \text{for } \sqrt{\omega_1^2+\omega_2^2} < \pi \\ \text{constant} & \text{rest } 0 \leq |\omega_1|,|\omega_2| \leq \pi \end{cases}$
$\rightarrow$ h[i,j]=h($\sqrt{i^2+j^2}$) (circularly symmetric)

But: $h[i,j]=h(\sqrt{i^2+j^2})$ does **NOT**
$\rightarrow H(e^{j\omega_1},e^{j\omega_2}) = H(\sqrt{\omega_1^2+\omega_2^2})$.