TOPICS FOR TODAY'S LECTURE

Infinite Impulse Response (IIR)
Filter Design and Examples

1. Impulse Invariance
2. Bilinear Transformation
   and Frequency Warping

WHY USE IIR FILTERS? [1/3]

Advantages:

1. Much sharper filters;
   less memory and MADs
2. Don't need to add delay
   to make h[n] causal
3. Apply analog filter lore

to: digital filter design

WHY USE IIR FILTERS? [2/3]

Disadvantages:

1. Not linear phase

so: phase distortion

2. Roundoff error can
   make filter unstable
3. *Know* analog filters?

WHY USE IIR FILTERS? [3/3]

But: Problem: #coefficients$\to \infty$
so: Can't optimize h[n] directly.

Given: Analog lowpass (or whatever) filter $H_a(s)$.
Goal: Digital lowpass (or whatever) filter $H(z)$.
Map: Substitute *mapping* s=function(z) in $H_a(s)$
Need: $Re[s] < 0 \to |z| < 1$ (stability preserved).

IMPULSE INVARIANCE

Given: Analog filter $H_a(s)$.

1. Compute $h_a(t)=\mathcal{L}^{-1}\{H_a(s)\}$.
2. h[n]=Th_a(nT) for constant T.

That is: Sample continuous-time $h_a(t)$.

Note: $h_a(t)=\sum_{i=1}^{N} A_i e^{p_i t} u(t)$

Becomes: h[n]=T$\sum_{i=1}^{N} A_i e^{(p_i T)n} u[n]$.
Map: Poles $\{p_i\} \to \{e^{p_i T}\}$.

IMPULSE INVARIANCE: EX #1 [1/2]

Given: $H_a(s)$=1000/(s+1000) and T=0.001.

Goal: Design digital IIR filter from $H_a(s)$
using: impulse invariance technique.

Soln: $h_a(t)=\mathcal{L}^{-1}\{H_a(s)\}=1000e^{-1000t}u(t)$.

Answer: h[n]=(0.001)(1000)$e^{-(1000)(0.001)n}=e^{-n}u[n]$.

IMPULSE INVARIANCE: EX #1 [2/2]

Why? Did I use T=0.001 in example above?

since: $|H(j\Omega)|=1000/\sqrt{\Omega^2+10^6}$.

so that: 3 dB freq=$1000\frac{\text{RADIAN}}{\text{SECOND}}$ ≈160 Hz.

suggests: Sample $1000\frac{\text{SAMPLE}}{\text{SECOND}}$ →T=0.001.

Note: H(z)=$\mathcal{Z}\{e^{-n}u[n]\}=\frac{z}{z-0.368}$. Lowpass.

IMPULSE INVARIANCE: EX #2

Given: $H_a(s)=\frac{s+0.1}{(s+0.1)^2+9}$. Poles:–0.1±j3.

Goal: Design digital IIR filter from $H_a(s)$
using: impulse invariance technique.

Soln: $h_a(t)=\mathcal{L}^{-1}\{H_a(s)\}=e^{-0.1t}\cos(3t)u(t)$.

Answer: h[n]=$Te^{-0.1nT}\cos(3nT)u[n]$.

BILINEAR TRANSFORMATION [1/4]

Given: Analog filter $H_a(s)$.

1. Substitute s=$\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}=\frac{2}{T}\frac{z-1}{z+1}$.
2. Simplify H(z)=$H_a\left(s=\frac{2}{T}\frac{z-1}{z+1}\right)$.

Note: "Bilinear"=ratio of linears.
Map: Preserves stability and
Map: $(s=j\Omega)\Leftrightarrow(z=e^{j\omega})$.

BILINEAR TRANSFORMATION [2/4]

Maps: Imaginary axis Re[s]=0 ⇔ s=$j\Omega$
to: Unit circle $|z|=1\Leftrightarrow z=e^{j\omega}$:

Note: s=$\frac{2}{T}\frac{z-1}{z+1}\Leftrightarrow z=(1+sT/2)/(1-sT/2)$.

s=$j\Omega$: $z=\frac{1+j\Omega T/2}{1-j\Omega T/2}=\frac{1+j\Omega T/2}{(1+j\Omega T/2)^*}\to|z|$=1. QED.

BILINEAR TRANSFORMATION [3/4]

Maps: Unit circle $|z|=1\Leftrightarrow z=e^{j\omega}$
to: Imaginary axis Re[s]=0 ⇔ s=$j\Omega$:

z=$e^{j\omega}$: s=$\frac{2}{T}\frac{e^{j\omega}-1}{e^{j\omega}+1}=\frac{2}{T}\frac{e^{j\omega/2}(e^{j\omega/2}-e^{-j\omega/2})}{e^{j\omega/2}(e^{j\omega/2}+e^{-j\omega/2})}$.

becomes: s=$j\frac{2}{T}\tan\left(\frac{\omega}{2}\right)$=j$\Omega$. QED.

BILINEAR TRANSFORMATION [4/4]

Frequency (Pre)Warping:
Bilinear transform maps frequencies:
(Pre)Warping formula: $\Omega=\frac{2}{T}\tan\left(\frac{\omega}{2}\right)$.

Use this to choose analog frequencies
to map to desired digital frequencies.

BILINEAR TRANSFORM: EX #1

Given: $H_a(s)$=1000/$(s+1000)$ and T=0.001.
Goal: Design digital IIR filter from $H_a(s)$
using: bilinear transformation technique.

Soln: Substitute s=$\frac{2}{T}\frac{z-1}{z+1}$ in $H_a(s)$.

Get: H(z)=$\frac{1000}{2000\frac{z-1}{z+1}+1000}=\frac{z+1}{3z-1}$. Lowpass.

BILINEAR: EX #2 [1/2]

Given: $H_a(s)=\frac{s+0.1}{(s+0.1)^2+16}$.
Note: Resonant peak at $\Omega = 4$.

Goal: Design digital IIR filter
with: Resonant peak at $\omega = \frac{\pi}{2}$.
using: bilinear transform.

Idea: Choose T so Ω=4$\rightarrow$ ω=$\frac{\pi}{2}$.

BILINEAR: EX #2 [2/2]

Use: **Prewarping:** $\Omega=\frac{2}{T}\tan\frac{\omega}{2}$.
Solve: $4=\frac{2}{T}\tan\frac{\pi}{4}=\frac{2}{T}\rightarrow T=\frac{1}{2}$.

Then: Substitute s=$4\frac{z-1}{z+1}$ in $H_a(s)$.

Get: H(z)=$\frac{0.128-0.122z^{-2}}{1+0.975z^{-2}}$.

Poles: $0.987e^{\pm j\pi/2}$ as desired.

BILINEAR: EX #3

Goal: Design one-pole digital filter
with: 3 dB bandwidth in ω of 0.2π.

Soln: **Prewarp:** $\Omega=\frac{2}{T}\tan(\frac{0.2\pi}{2})=\frac{0.65}{T}$.
Then: $H_a(s)=\frac{0.65/T}{s+0.65/T}$ (one-pole analog).

Get: H(z)=$\frac{0.245(z+1)}{z-0.509}$. T cancels!

Check: $|H(e^{j0.2\pi})| = 0.707$.

MATLAB COMMANDS

$H_a(s)$: Numerator `BC` & denominator `AC`.
$H(z)$: Numerator **BD** & denominator `AD`.
Then: `[BD,AD]=bilinear(BC,AC,F);`
substitutes $s=2F\frac{z-1}{z+1}$ in $H_a(s)$.

Want: Butterworth filter: order=N
Cutoff: frequency: W (Digital:0≤W≤1.)
`[B,A]=butter(N,W);` (digital)
`[B,A]=butter(N,W,'s');` (analog)

MATLAB'S FDATOOL GUI