TOPICS FOR TODAY'S LECTURE

Finite Impulse Response (FIR)
Filter Design and Examples

0. Linear phase-Why it's good
1. Windowing ideal filters
2. Frequency Sampling
3. Equiripple (Matlab)

WHY USE FIR FILTERS? [2/2]

Disadvantages:

1. May need *long* duration h[n].
2. IIR h[n] more selective with less memory and fewer MADs.

Note: FIR=MA (Moving Average).

WHY USE LINEAR PHASE? [2/3]

So: $h[n]$ has linear phase means that
$\leftrightarrow$ h[n]=g[n–D] where g[n]=±g[–n].
Then: $\arg[H(e^{j\omega})]=\frac{\pi k}{2}-\omega D$ (linear phase)
where: k=one of $\{0,1,2,3\}$ (so affine phase)
and: $|H(e^{j\omega})|=|G(e^{j\omega})|$ (have same gain).

WHY USE FIR FILTERS? [1/2]

Form: y[n]=b_0x[n]+b_1x[n–1]+…+b_Nx[n–N].

Advantages:

1. Always stable (poles at origin).
2. Make causal using finite delay.
3. Finite duration→transient=0 after N time samples (unlike IIR).

WHY USE LINEAR PHASE? [1/3]

DTFT: h[n] even→phase=$\{0,\pi\}$.
DTFT: h[n] odd→phase=$\{\frac{\pi}{2},-\frac{\pi}{2}\}$.
So: Alter magnitudes, not phase.
Note: 4^{th} advantage of FIR filters.

WHY USE LINEAR PHASE? [3/3]

EX: Linear phase lowpass filter with cutoff ω_c.
DTFTs: $Y(e^{j\omega})=H(e^{j\omega})X(e^{j\omega})$.
$|\omega|<\omega_c$: $Y(e^{j\omega})=X(e^{j\omega})e^{-j\omega D}$
Effect: In passband, y[n]=x[n–D] (no distortion).
$|\omega|>\omega_c$: $Y(e^{j\omega})\approx 0$
Effect: In stopband, phase distortion, but y[n]=0.

FORMS OF LINEAR PHASE

Type	FORM OF h[n]	APPLICATION	**Restriction**
I	$\{b,a,c,a,b\}$	BandReject	None
II	$\{b,a,a,b\}$	LowPass only	$H(e^{j\pi})=0$
III	$\{b,a,0,-a,-b\}$	BandPass only	$H(e^{j0,j\pi})=0$
IV	$\{b,a,-a,-b\}$	HighPass	$H(e^{j0})=0$

No: You don't have to memorize "Types I-IV"!
Zeros: Complex conjugate reciprocal quadruples:
Means: $\{z_o, z_o^*, \frac{1}{z_o}, \frac{1}{z_o^*}\}$. Note $z_o^*=\frac{1}{z_o}$ on unit circle.

ALL-PASS FILTERS

What: $|H(e^{j\omega})|=1$ but $\arg[H(e^{j\omega})]\neq 0$.
Why? Correct phase distortion for IIR.

Need: Zeros and poles in reciprocal pairs:
That is: $\exists$ zero $z_o \rightarrow \exists$ pole $1/z_o^*$
and: $\exists$ pole $p_o \rightarrow \exists$ zero $1/p_o^*$.

H(z) $=\frac{1}{z_o^*}\frac{z-z_o}{z-1/z_o^*} \rightarrow H(z)H^*(1/z^*)=1$.

WINDOWING IDEAL h[n] [1/2]

Idea: h[n]=h_{IDEAL}[n]w[n] for some
data window (Hamming) w[n].

Matlab: `h=fir1(N-1,W,'ftype',window);`
where: W=vector of passband cutoffs.

EX: 5-point digital differentiator
using a rectangular window:

WINDOWING IDEAL h[n] [2/2]

Soln: $h_{\text{IDEAL}}[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}(j\omega)e^{j\omega n}d\omega$
$=\frac{(-1)^n}{n}=\{\ldots-\frac{1}{2},1,\underline{0},-1,\frac{1}{2}\ldots\}$

Note: Inverse DTFT$\left\{\begin{matrix}\text{IMAG}\\ \text{ODD}\end{matrix}\right\}=\left\{\begin{matrix}\text{REAL}\\ \text{ODD}\end{matrix}\right\}$.

Answer: h[n]=h_{IDEAL}[n]w[n]=$\{-\frac{1}{2},1,\underline{0},-1,\frac{1}{2}\}$.
$\rightarrow$ y[n]=$-\frac{1}{2}$x[n+2]+x[n+1]–x[n–1]+$\frac{1}{2}$x[n–2].
Compare: To (3-point) central difference operator.

FREQUENCY SAMPLING [1/3]

Solve: $\sum_{n=-N/2}^{N/2} h[n]e^{-j\omega_k n}=H_D(e^{j\omega_k})$
for: frequencies ω_k (usually $\omega_k=\frac{2\pi k}{N+1}$)
and: $H_D(e^{j\omega})$=Desired frequency response.

EX: Design a 5-point digital lowpass filter.
Using: Type I linear phase filter and specs:
Desire: $H_D(e^{j0})=1$. $H_D(e^{j\frac{\pi}{2}})=\frac{3}{4}$. $H_D(e^{j\pi})=0$.

FREQUENCY SAMPLING [2/3]

Soln: Type I: h[n]=$\{a,b,\underline{c},b,a\}$.
$H(e^{j\omega})$ =c+2bcos(ω)+2acos(2ω).
ω=0: c+2b+2a=1=$H(e^{j0})$.
$\omega=\pi$: c—2b+2a=0=$H(e^{j\pi})$.
$\omega=\pi/2$: c—2a $=\frac{3}{4}=H(e^{j\frac{\pi}{2}})$.

FREQUENCY SAMPLING [3/3]

Answer: h[n]=$\{a, b, \underline{c}, b, a\}$
$=\{-\frac{1}{16}, \frac{1}{4}, \frac{5}{8}, \frac{1}{4}, -\frac{1}{16}\}$.

OR: `ifft([1 3/4 0 3/4])`
`=[5/8 1/4 -1/8 1/4].`
$=$*aliased* h[n]: $\frac{1}{16}+\frac{1}{16}=\frac{1}{8}$.

Matlab: `h=fir2(N-1,F,M,window)`
where: M=magnitudes at freqs=F.

EQUIRIPPLE FILTER DESIGN [1/3]

Goal: $\underset{\text{h[n]}}{\text{MIN}}\ \underset{\omega}{\text{MAX}} \left\{|E(e^{j\omega})|W(e^{j\omega})\right\}$ (minimax)
where: $E(e^{j\omega})=H_D(e^{j\omega}) - \sum_{n=0}^{N-1} h[n]e^{-j\omega n}$.

Huh? Minimize largest error (worst case)
so: |error|=$|E(e^{j\omega})|$ is never very large.

Solution: Iterative algorithm (Parks & McClellan)
using: Remez exchange and alternation theorems.

EQUIRIPPLE FILTER DESIGN [2/3]

Matlab: `h=firpm(N-1,F,M,weights,ftype)`
where: M=magnitudes at frequencies=F
and: F=vector of $\frac{\omega}{\pi}$: Max freq $\omega=\pi \rightarrow$ F=1.
Also: ftype=filter type (omit for Types I,II)
and: ftype='hilbert' (use this for Types III,IV).

EQUIRIPPLE FILTER DESIGN [3/3]

Weights: $W(e^{j\omega}) = \begin{cases} \delta_1 & \text{for } \omega > \omega_s \text{ (stopband)} \\ \delta_2 & \text{for } \omega < \omega_p \text{ (passband)} \end{cases}$
Why: Control relative ripple sizes in bands.
Also: ftype='differentiator': $W(e^{j\omega})=\frac{1}{\omega}$
vs.: ftype='hilbert': $W(e^{j\omega})=1$.
since: Want to penalize errors near ω=0.

EQUIRIPPLE EXAMPLE [1/2]

EX: Design an FIR lowpass filter
using: Equiripple design procedure.
Specs: Length=61 (so order=60)
Passband: $0 \leq |\omega| < 0.2\pi$
Transition: $0.2\pi < |\omega| < 0.3\pi$
Stopband: $0.3\pi < |\omega| \leq \pi$

EQUIRIPPLE FILTER DESIGN: EX [2/2]

Soln: `firpm(60,[0.0,0.2,0.3,1.0],[1,1,0,0])`
Note: Magnitudes and frequencies are always specified in pairs defining *bands* of specs.
Compare: See 2^{nd} ed. text p. 648-650: has answer.