TOPICS FOR TODAY’S LECTURE

- Finite Impulse Response (FIR)
- Filter Design and Examples

0. Linear phase—Why it’s good
1. Windowing ideal filters
2. Frequency Sampling
3. Equiripple (Matlab)

WHY USE FIR FILTERS? [1/2]

Form: \[ y[n]=b_0x[n]+b_1x[n-1]+\ldots+b_Nx[n-N]. \]

Advantages:
1. Always stable (poles at origin).
3. Finite duration—transient=0 after N time samples (unlike IIR).

WHY USE FIR FILTERS? [2/2]

Disadvantages:
1. May need long duration \( h[n] \).
2. IIR \( h[n] \) more selective with less memory and fewer MADs.

Note: FIR=MA (Moving Average).

WHY USE LINEAR PHASE? [1/3]

DTFT: \( h[n] \) even \( \rightarrow \) phase=\{0, \pi\}.
DTFT: \( h[n] \) odd \( \rightarrow \) phase=\{\pi/2, -\pi/2\}.

So: Alter magnitudes, not phase.

Note: \( 4^{th} \) advantage of FIR filters.

WHY USE LINEAR PHASE? [2/3]

So: \( h[n] \) has linear phase means that
\[ h[n]=g[n-D] \] where \( g[n]=\pm g[-n] \).

Then: \[ \arg[H(e^{j\omega})]=\frac{\pi k}{2} - \omega D \] (linear phase)
where: \( k=\)one of \( \{0,1,2,3\} \) (so affine phase)
and: \[ |H(e^{j\omega})|=|G(e^{j\omega})| \] (have same gain).

WHY USE LINEAR PHASE? [3/3]

EX: Linear phase lowpass filter with cutoff \( \omega_c \).

DTFTs: \[ Y(e^{j\omega})=H(e^{j\omega})X(e^{j\omega}). \]
\[ |\omega|<\omega_c: \ Y(e^{j\omega})=X(e^{j\omega})e^{-j\omega D} \]

Effect: In passband, \( y[n]=x[n-D] \) (no distortion).
\[ |\omega|>\omega_c: \ Y(e^{j\omega})\approx 0 \]

Effect: In stopband, phase distortion, but \( y[n]=0 \).
FORMS OF LINEAR PHASE

<table>
<thead>
<tr>
<th>Type</th>
<th>Form of $h[n]$</th>
<th>Application</th>
<th>Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>${b, a, c, a, b}$</td>
<td>BandReject</td>
<td>None</td>
</tr>
<tr>
<td>II</td>
<td>${b, a, a, b}$</td>
<td>LowPass only</td>
<td>$H(e^{j\pi}) = 0$</td>
</tr>
<tr>
<td>III</td>
<td>${b, a, 0, -a, -b}$</td>
<td>BandPass only</td>
<td>$H(e^{j0, j\pi}) = 0$</td>
</tr>
<tr>
<td>IV</td>
<td>${b, a, -a, -b}$</td>
<td>HighPass</td>
<td>$H(e^{j0}) = 0$</td>
</tr>
</tbody>
</table>

Note: You don’t have to memorize “Types I-IV”!

Zeros: Complex conjugate reciprocal quadruples:

$\{z_0, z_0^*, \frac{1}{z_0}, \frac{1}{z_0^*}\}$. Note $z_0^* = \frac{1}{z_0}$ on unit circle.

ALL-PASS FILTERS

What: $|H(e^{j\omega})|=1$ but $\arg[H(e^{j\omega})]\neq 0$.

Why? Correct phase distortion for IIR.

Need: Zeros and poles in reciprocal pairs:

That is: $\exists$ zero $z_0 \rightarrow \exists$ pole $1/z_0^*$

and: $\exists$ pole $p_0 \rightarrow \exists$ zero $1/p_0^*$.

$H(z) = \frac{1 - z^{-N}}{z^{-1} - 1/z_0^*} \rightarrow H(z)H^*(1/z^*) = 1$.

WINDOWING IDEAL $h[n]$ [1/2]

Idea: $h[n] = h_{\text{IDEAL}}[n]w[n]$ for some data window (Hamming) $w[n]$.

Matlab: $h = \text{fir1}(N-1,W,'ftype', \text{window})$;
where: $W =$ vector of passband cutoffs.

EX: 5-point digital differentiator using a rectangular window:

Answer: $h[n] = h_{\text{IDEAL}}[n]w[n] = \{-\frac{1}{2}, 1, 0, -1, \frac{1}{2}\}$.

Compare: To (3-point) central difference operator.

WINDOWING IDEAL $h[n]$ [2/2]

Solve: $\sum_{n=0}^{N/2} h[n]e^{-j\omega_n} = H_D(e^{j\omega})$
for: frequencies $\omega_k$ (usually $\omega_k = \frac{2\pi k}{N+1}$)
and: $H_D(e^{j\omega})=$ Desired frequency response.

EX: Design a 5-point digital lowpass filter.
Using: Type 1 linear phase filter and specs:
Desire: $H_D(e^{j0}) = 1$, $H_D(e^{j\pi}) = \frac{3}{4}$, $H_D(e^{j2\pi}) = 0$.

FREQUENCY SAMPLING [1/3]

Solve: $\sum_{n=-N/2}^{N/2} h[n]e^{-j\omega_n} = H_D(e^{j\omega})$
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FREQUENCY SAMPLING [2/3]

Solve: $\sum_{n=-N/2}^{N/2} h[n]e^{-j\omega_n} = H_D(e^{j\omega})$
for: frequencies $\omega_k$ (usually $\omega_k = \frac{2\pi k}{N+1}$)
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Note: Inverse DTFT $\{\text{IMAG} \} = \{\text{REAL} \}$.

Answer: $h[n] = h_{\text{IDEAL}}[n]w[n] = \{-\frac{1}{2}, 1, 0, -1, \frac{1}{2}\}$.

Compare: To (3-point) central difference operator.
FREQUENCY SAMPLING [3/3]

Answer: \( h[n] = \{a, b, c, b, a\} \)
= \(-\frac{1}{16}, \frac{1}{4}, \frac{5}{8}, \frac{1}{4}, -\frac{1}{16}\).

\[ \text{OR: } \text{ifft([1 3/4 0 3/4])} \]
= \([5/8 \ 1/4 \ -1/8 \ 1/4]\).
= aliased \( h[n] \): \( \frac{1}{16} + \frac{1}{16} = \frac{1}{8} \).

Matlab: \( h = \text{fir2}(N-1,F,M,\text{window}) \)
where: \( M = \text{magnitudes at freqs=F} \).

EQUIRIPPLE FILTER DESIGN [1/3]

Goal: \( \min \max_{\omega} \left\{ |E(e^{j\omega})|W(e^{j\omega}) \right\} \) (minimax)
where: \( E(e^{j\omega}) = H_D(e^{j\omega}) - \sum_{n=0}^{N-1} h[n]e^{-j\omega n} \).

Huh? Minimize largest error (worst case)
so: \( |\text{error}| = |E(e^{j\omega})| \) is never very large.

Solution: Iterative algorithm (Parks & McClellan)
using: Remez exchange and alternation theorems.

EQUIRIPPLE FILTER DESIGN [2/3]

Matlab: \( h = \text{firpm}(N-1,F,M,\text{weights,ftype}) \)
where: \( M = \text{magnitudes at frequencies=F} \)
and: \( F = \text{vector of \( \frac{\omega}{\pi} \): Max freq \( \omega=\pi \rightarrow F=1 \).} \)
Also: \( \text{ftype=filter type (omit for Types I,II)} \)
and: \( \text{ftype='hilbert' (use this for Types III,IV).} \)

EQUIRIPPLE FILTER DESIGN [3/3]

Weights: \( W(e^{j\omega}) = \begin{cases} \delta_1 & \text{for } \omega > \omega_s \text{ (stopband)} \\ \delta_2 & \text{for } \omega < \omega_p \text{ (passband)} \end{cases} \)

Why: Control relative ripple sizes in bands.
Also: \( \text{ftype='differentiator': } W(e^{j\omega}) = \frac{1}{\omega} \)
vs: \( \text{ftype='hilbert': } W(e^{j\omega}) = 1 \).
since: Want to penalize errors near \( \omega=0 \).

EQUIRIPPLE EXAMPLE [1/2]

EX: Design an FIR lowpass filter
using: Equiripple design procedure.
Specs: Length=61 (so order=60)
Passband: \( 0 \leq |\omega| < 0.2\pi \)
Transition: \( 0.2\pi < |\omega| < 0.3\pi \)
Stopband: \( 0.3\pi < |\omega| \leq \pi \)

EQUIRIPPLE FILTER DESIGN: EX [2/2]

Soln: \( \text{firpm}(60,[0.0,0.2,0.3,1.0],[1,1,0,0]) \)
Note: Magnitudes and frequencies are always specified in pairs defining bands of specs.
Compare: See 2nd ed. text p. 648-650: has answer.