FIR: Finite Impulse Response: $h[n]$ has finite duration. All poles at origin.
MA: Moving Average filter: y[n]=b_0x[n]+b_1x[n-1]+…+b_Nx[n-N] (order=N).
Why? Always stable; add delay to make causal; easy to design (see below).

LINEAR PHASE:

TYPE	FORM OF h[n]	APPLICATION	RESTRICTION
I	$\{b,a,c,a,b\}$	Low or highpass	None
II	$\{b,a,a,b\}$	Lowpass only	$H(e^{j\pi})=0$
III	$\{b,a,0,-a,-b\}$	Differentiator	$H(e^{j0})=H(e^{j\pi})=0$
IV	$\{b,a,-a,-b\}$	Differentiator	$H(e^{j0})=0$

Why? No phase distortion (just time delay); affects only Fourier magnitudes.
Zeros: Complex conjugate reciprocal quadruples: $\{z_o, z_o^*, \frac{1}{z_o}, \frac{1}{z_o^*}\}$. Coincide?

THREE FIR FILTER DESIGN TECHNIQUES

WINDOW DESIGN: h[n]= h_{IDEAL}[n]w[n] for some **data window** (e.g., Hamming) w[n].

Example: Design a 5-point digital differentiator using a rectangular window.
$h_{IDEAL}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}(j\omega)e^{j\omega n}d\omega = \frac{(-1)^n}{n} = \{\ldots -\frac{1}{2}, 1, \underline{0}, -1, \frac{1}{2}\ldots\}$. IMAG ODD $\rightarrow$ REAL EVEN.

Solution: $h[n]=\{-\frac{1}{2},1,\underline{0},-1,\frac{1}{2}\}\rightarrow$ y[n]=$-\frac{1}{2}$x[n+2]+x[n+1]–x[n-1]+$\frac{1}{2}$x[n-2].

Matlab: `h=fir1(N-1,W,'ftype',window)`; W=vector of passband cutoffs.

FREQUENCY SAMPLING: Solve $\sum_{n=0}^{N-1} h[n]e^{-j\omega_k n} = H_{DESIRED}(e^{j\omega_k})$ for some frequencies ω_k.

Example: Design a 5-point digital lowpass filter using frequency sampling.
Constraints: Type I filter. $H(e^{j0})=1$. $H(e^{j\pi/2})=3/4$. $H(e^{j\pi})=0$. (Lowpass)

Solution: h[n]=$\{a,b,c,b,a\}$. $H(e^{j\omega})$=c+2bcos(ω)+2acos(2ω)=DTFT$\{h[n]\}$.
$\underline{\omega=0}$: c+2b+2a=1. $\underline{\omega=\pi}$: c–2b+2a=0. $\underline{\omega=\pi/2}$: c–2a=3/4.
Answer: Solving gives: h[n]=$\{a,b,\underline{c},b,a\}=\{-\frac{1}{16},\frac{1}{4},\overline{\frac{5}{8},\frac{1}{4},-\frac{1}{16}}\}$.

Alternate: `ifft([1 3/4 0 3/4])=[5/8 1/4 -1/8 1/4]`=*aliased* h[n]: $\frac{1}{16}+\frac{1}{16}=\frac{1}{8}$.

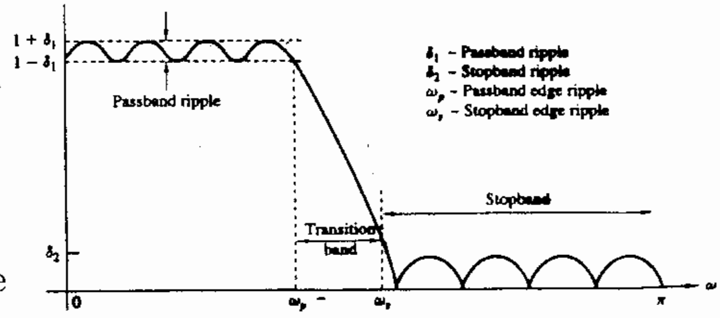
Matlab: `h=fir2(N-1,F,M,window)` where M=magnitudes at frequencies=F.

EQUI–RIPPLE: $\underset{h[n]}{\text{MIN}}\ \underset{\omega}{\text{MAX}}|E(e^{j\omega})|W(e^{j\omega}) = \underset{h[n]}{\text{MIN}}\ \underset{\omega}{\text{MAX}} W(e^{j\omega})|H_D(e^{j\omega})-\sum_{n=0}^{N-1}h[n]e^{-j\omega_n}|$
Huh? Minimize largest error: No frequency far off. Worst case minimized.

Solution: Iterative algorithm using Remez exchange and alternation theorems.
Matlab: `h=firpm(N-1,F,M,weights,ftype)` M=magnitudes at frequencies=F.
where: F=vector of $\frac{\omega}{\pi}$: Maximum frequency ω=π $\rightarrow$ F=1. F includes 0,1.

Goal: Design FIR (Finite Impulse Response) linear phase filter.
How? (1) Specify design criteria;
(2) Choose $h(n), n = 0 \ldots M$ to minimize cost criterion.

Want linear phase→no phase distortion–only magnitude changes.



Magnitude characteristics of physically realizable filters.

Linear Phase: $h(n) = \pm h(M-n) \to H(z) = \pm z^{-M} H(1/z)$. Don't need p. 621-623!
Zeros in complex-conjugate quadruples: $\{z_i, z_i^*, 1/z_i, 1/z_i^*\}$.
If $|z_i| = 1$ (zero on unit circle) then $1/z_i^* = z_i$ in quadruples.
If $h(n) = -h(M-n)$ then $H(e^{j0}) = \sum h(n) = 0$–can't use for LPF!
But want this for odd funcs, e.g., differentiator, Hilbert transform.

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1. **Frequency Sampling:** $h(n) = \frac{1}{M} \sum_{k=0}^{M-1} H(e^{j2\pi k/M}) e^{j2\pi nk/M}$.

Interpretation: Just inverse DFT of desired freq. response sampled at $\omega = \frac{2\pi}{M} k$.
$h(n) = \pm h(M-1-n) \to$simplifications (sin and cos) on p. 633.
Problem: In between the specified $H(e^{j2\pi k/M})$, anything can happen.

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2. **Equiripple Design:** Weight $W(\omega) = \begin{cases} \delta_1 & \text{for } \omega > \omega_s \text{ (stopband)} \\ \delta_2 & \text{for } \omega < \omega_p \text{ (passband)} \end{cases}$.

Cost: $\min_{h(n)} \left[\max_{\omega} \left[W(\omega) |H_{desired}(\omega) - \sum_{n=0}^{M-1} h(n) e^{-j\omega n}| \right] \right]$. Note: $|error|$.

Interpretation: Minimax criterion–"playing a game with nature, who is out to get us."
Actually, use $h(n) = \pm h(M-1-n)$ to reduce #unknowns (p. 638-641).

Sol'n: Parks and McClellan used *Chebyschev approximation* theory in 1972.
Result: *Alternation thm.* →many ripples in both stopband and passband.
Solve: *Remez exchange* algorithm: iterative procedure (flowchart p. 647).

Matlab: Command **remez(60,[0.0,0.2,0.3,1.0],[1,1,0,0])** computes $h(n)$ for a lowpass filter with length 61 (sic) with $\omega_p = 0.2\pi$ and $\omega_s = 0.3\pi$.
Bands: $0 < \omega < 0.2\pi$ (passband) and $0.3\pi < \omega < \pi$ (stopband). 1.0=Nyquist.

This problem is solved on p. 648-650; try this and compare the $h(n)$.
vector1: Frequencies in pairs that define bands in which $|H(e^{j\omega})|$ is specified.
vector2: Amplitudes between which $|H(e^{j\omega})|$ varies linearly in the bands.

Goal: Design a 7-point digital differentiator using:
(a) Rectangular window; (b) Hamming window;
(c) Frequency sampling; (d) Inverse DFT.

(a),(b) $h[n]=h_{\text{IDEAL}}[n]w[n]$ where $w[n]$=the window and:
$h_{\text{IDEAL}}[n]$=inverse DTFT of $H_{\text{DESIRED}}(e^{j\omega})$.

Here: $H_{\text{DESIRED}}(e^{j\omega})=j\omega$ for $|\omega|<\pi$ since $H_a(j\Omega)=j\Omega$.
Then: $h_{\text{IDEAL}}[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}(j\omega)e^{j\omega n}d\omega=\frac{(-1)^n}{n}$. Also, $h_{\text{IDEAL}}[0]=0$.
$h_{\text{IDEAL}}[n]=\{\dots\frac{1}{5},-\frac{1}{4},\frac{1}{3},-\frac{1}{2},1,\underline{0},-1,\frac{1}{2},-\frac{1}{3},\frac{1}{4},-\frac{1}{5}\dots\}$
Note: $H_{\text{DESIRED}}(e^{j\omega})$ imaginary and odd$\Leftrightarrow h_{\text{IDEAL}}[n]$ real and odd.

(a) $w[n]=\{\dots 0,0,1,1,1,\underline{1},1,1,1,0,0\dots\}$ (7-point rectangular window).
$h[n]=h_{\text{IDEAL}}[n]w[n]=\{\frac{1}{3},-\frac{1}{2},1,\underline{0},-1,\frac{1}{2},-\frac{1}{3}\}$.

(b) $w[n]=0.54+0.46\cos(\pi\frac{n}{3}),|n|\leq 3=\{.08,.31,.77,\underline{1},.77,.31,.08\}$.
Note: $w[n]$ goes from 0 to 6: Use $w[n]=0.54-0.46\cos(\pi\frac{n}{3})$.
Here: $w[n]$ goes from –3 to 3: Use $w[n]=0.54+0.46\cos(\pi\frac{n}{3})$.

$h[n]=h_{\text{IDEAL}}[n]w[n]=\{\frac{.08}{3},-\frac{.31}{2},0.77,\underline{0},-0.77,\frac{.31}{2},-\frac{.08}{3}\}$.
$h[n]=\{0.0267,-0.155,0.77,0,-0.77,0.155,-0.0267\}$.

(c) Solve $\sum_{n=-3}^{3}h[n]e^{-j\omega n}=j\omega$ for 7 values of ω from $-\pi$ to $+\pi$:
$\omega=2\pi\frac{k}{7},|k|\leq 3=\{-\frac{6}{7}\pi,-\frac{4}{7}\pi,-\frac{2}{7}\pi,0,\frac{2}{7}\pi,\frac{4}{7}\pi,\frac{6}{7}\pi\}$. Use $h[-n]=-h[n]$.
Note: Don't include $\pm\pi$: $H(e^{j\omega})$ is discontinuous at $\pm\pi$! Jumps $j\pi$ to $-j\pi$.

Then: $\sum_{n=-3}^{3}h[n]e^{-j\omega n}=\sum_{n=1}^{3}(h[-n]e^{j\omega n}+h[n]e^{-j\omega n})$
$=\sum_{n=1}^{3}h[n](e^{-j\omega n}-e^{j\omega n})=-2j\sum_{n=1}^{3}h[n]\sin(\omega n)=j\omega$.

$$-2\begin{bmatrix}\sin(\frac{2}{7}\pi) & \sin(\frac{4}{7}\pi) & \sin(\frac{6}{7}\pi)\\ \sin(\frac{4}{7}\pi) & \sin(\frac{8}{7}\pi) & \sin(\frac{12}{7}\pi)\\ \sin(\frac{6}{7}\pi) & \sin(\frac{12}{7}\pi) & \sin(\frac{18}{7}\pi)\end{bmatrix}\begin{bmatrix}h[1]\\ h[2]\\ h[3]\end{bmatrix}=\begin{bmatrix}\frac{2}{7}\pi\\ \frac{4}{7}\pi\\ \frac{6}{7}\pi\end{bmatrix}.$$

This has the solution $h[1]$=–1.03; $h[2]$=0.574;$h[3]$=–0.46 using

```
A=-2*sin(pi/7*2*[1 2 3]'*[1 2 3]);B=pi/7*2*[1 2 3]';A\B
```

Then $h[n]=\{0.46,-0.574,1.03,0,-1.03,0.574,-0.46\}$.

(d) Take the periodic extension of $h[n]$:
$\{\dots h[-3],h[-2],h[-1],\underline{h[0]},h[1],h[2],h[3],h[-3],h[-2],h[-1],h[0]\dots\}$
This has DTFS $h_k=\frac{1}{7}\sum_{n=-3}^{3}h[n]e^{-j2\pi nk/7}$ (use any period of $h[n]$).

But: This is what we get setting $\omega=2\pi\frac{k}{7},|k|\leq 3$ in $\sum_{n=-3}^{3}h[n]e^{-j\omega n}$.
So: $\sum_{n=-3}^{3}h[n]e^{-j\omega n}=j\omega,|\omega|=2\pi\frac{k}{7}=\Leftrightarrow 7h_k=2\pi\frac{k}{7},|k|\leq 3$.

DFT: $H_k=\sum_{n=0}^{7-1}h[n]e^{-j2\pi nk/7}=7h_k$ for k=0,1,2,3,4,5,6.
where: $h_k=h_{k-7}$ for $k=4,5,6$. So use $\{h_0,h_1,h_2,h_3,h_{-3},h_{-2},h_{-1}\}$.

So: `real(ifft(j*pi/7*[0 2 4 6 -6 -4 -2]))`
Gives: `[0,-1.03,0.574,-0.46,0.46,-0.574,1.03]`
This is: $\{h[0],h[1],h[2],h[3],h[-3],h[-2],h[-1]\}$.