ASSIGNED: Apr. 02, 2015. **READ:** Sects. 10.3 & 11.1-11.4.
DUE DATE: Apr. 09, 2015. **TOPICS:** FIR and IIR filter design.

Please [box] your answers. Show your work. Turn in all Matlab plots and Matlab code.

[20] 1. **IIR filter design of a digital integrator**. Let $H_a(s)=\frac{1}{s}$ and $T=2$.

- [5] (a) Compute $H_a(j\Omega)$ of the *analog* integrator $H_a(s)$. Compute $|H_a(j0)|$ and $|H_a(j\infty)|$.
- [5] (b) Use bilinear transform to design a *digital* integrator $H(z)$ & its difference equation.
- [5] (c) Compute $H(e^{j\omega})$ of the *digital* integrator $H(z)$. Compute $|H(e^{j0})|$ and $|H(e^{j\pi})|$. Express $H(e^{j\omega})$ in terms of $\sin(\omega/2)$ and $\cos(\omega/2)$ by factoring out $\frac{e^{j\omega/2}}{e^{j\omega/2}}$.
- [5] (d) Compare your answers to (a) and (c) for small ω and small Ω.

[25] 2. **IIR filter design of a 1-pole filter**. Let $H_a(s)=\frac{a}{s+a}$ and $T=2$.

- [5] (a) Compute $H_a(j\Omega)$ of the *analog* 1-pole filter $H_a(s)$. Compute $|H_a(j0)|$ and $|H_a(j\infty)|$.
- [5] (b) Use bilinear transform to design a *digital* 1-pole filter $H(z)$ (it will have a zero).
- [5] (c) Compute $H(e^{j\omega})$ of the *digital* 1-pole filter. Compute $|H(e^{j0})|$ and $|H(e^{j\pi})|$. Express $H(e^{j\omega})$ in terms of $\sin(\omega/2)$ and $\cos(\omega/2)$ by factoring out $\frac{e^{j\omega/2}}{e^{j\omega/2}}$.
- [5] (d) Compare your answers to (a) and (c) for small ω and small Ω.
- [5] (e) Let $H_a(s)=\frac{b}{s+b}$ and T=2. Choose b so that H(z) has its pole at $-a$.

[15] 3. **FIR filter design of a digital differentiator**. Let $H_a(s)=s$.

- [5] (a) Using MATLAB, plot the gain $|H(e^{j\omega})|$ of a digital differentiator of length=21 designed using the (Hamming) *window method*. For $h_{\text{IDEAL}}[n]$, see lecture.
- [5] (b) Using MATLAB, plot the gain $|H(e^{j\omega})|$ of a digital differentiator of length=21 designed using *frequency sampling* with $|H(e^{j\omega})|=|\omega|$ for $\omega = 2\pi\frac{k}{21}$ for $|k| \leq 10$.
- [5] (c) Compare (a) and (b) to the gain of the ideal differentiator $|H_a(j\omega)|=|j\omega|=|\omega|$. Plot both gains $|H(e^{j\omega})|$ for $0 \leq \omega < 2\pi$ (i.e., don't use MATLAB's `fftshift`).

[5] 4. **FIR filter design of a digital differentiator**.

Use `firpm` to design a digital differentiator of length=60.
Specs: Passband: $0 \leq \omega \leq 0.2\pi$. Stopband: $0.3\pi \leq \omega \leq \pi$.
Hint: Your answer should agree with page 654 of the 1997 ed of your text.
Note: Your answer should consist of *just a single MATLAB command*.

[15] 5. Download `p9.mat`. In MATLAB, type >>`load p9.mat` to get the sampled signal `Y`.

- [5] (a) Plot `Y` and its spectrum. There should be some spikes in the latter.
- [5] (b) `Y` came from `Y=filter([1],[1 zeros(1,N) 0.99],X);` From the location of the spikes in the spectrum, determine `N`.
- [5] (c) Recover `X` from `Y` using `X=filter([1 zeros(1,N) 0.99],[1],Y);` **Hint:** $|x[n]| \leq 1$ everywhere. We will analyze `X` in #6 below.

Put the 2 plots from #3 and the 2 from #5 in a (2×2) array using `subplot`.

[20] 6. Segment `X` into 8 segments of length 128 each (`X(1:128)`,`x(129:256)` etc.) Compute the DFT of each segment using a Hamming window. Plot the 1^{st} 64 points. Describe what the signal is in terms of how its spectrum changes over time.

Put the 8 plots for this problem in a (3×3) array using `subplot`.

"An elephant is a mouse built to goverment specifications."