
Please box your answers. Show your work. Turn in all Matlab plots and Matlab code.

[20] 1. We are given \( x[n] = \{ \ldots 18, 12, 6, 0, 6, 12, 18, 12, 6, 0, 6, 12, 18 \ldots \} \) (a periodic signal).

   [10] (a) Compute the Discrete-Time Fourier Series (DTFS) expansion of \( x[n] \).
   \textbf{Hint:} \( x[n] \) has period=6, \( x[n] \) is real-valued, and \( x[n] \) is an even function.
   [05] (b) Compute average power in the time domain. Your answer should agree with:
   [05] (c) Compute average power in the frequency domain.

[20] 2. We are given DTFS coefficients \( x_k = \cos(\pi k/4) + \sin(3\pi k/4) \).

   Note that \( x_k \) is entirely real-valued, but it is \textit{not} an even function.

   [5] (a) Explain why \( x[n] \) with these coefficients \( x_k \) has period=8.
   [5] (b) Compute the \textit{purely real} part of \( x[n] \) from the \textit{even} part of \( x_k \).
   [5] (c) Compute the \textit{imaginary} part of \( x[n] \) from the \textit{odd} part of \( x_k \).

\textbf{Hint:} One period of \( x[n] \) has only two real and two imaginary nonzero values.

\textbf{Hint:} Read off \( x[n] \) from \( x_k = \sum_{n=0}^{N-1} \frac{x[n]}{N} e^{-j2\pi nk/N} \). Note \( e^{-j2\pi nk/N} = e^{j2\pi (N-n)k/N} \).

[10] 3. Compute DTFTs of the following. Simplify to sums of sines and cosines.

   [5] (a) \( \{1, 1, \frac{1}{2}, 1, 1\} \). [5] (b) \( \{3, 2, 1\} \).

[10] 4. Use the inverse DTFT to evaluate the following integrals:

   [5] (a) \( \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{j\omega} e^{-j3\omega}}{e^{j\omega} - e^{-j\omega}} d\omega \).
   [5] (b) \( \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j3\omega} 2 \cos(3\omega) d\omega \).

   \textbf{Hint:} Replace \( e^{j\omega} \) with \( z \) and compute \( Z^{-1} \) at a specific \( n \).

[20] 5. \( x[n] = \{1, 4, 3, 2, 5, 7, -45, 7, 5, 2, 3, 4, 1\} \). Let \( X(e^{j\omega}) = \text{DTFT}[x[n]] \). Compute:

   [5] (a) \( X(e^{j\pi}) \). [5] (b) \( \arg[X(e^{j\omega})] \).
   [5] (c) \( \int_{-\pi}^{\pi} X(e^{j\omega}) d\omega \).
   [5] (d) \( \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega \).

   \textbf{Hint:} Do not actually compute \( X(e^{j\omega}) \)! Use some properties of the DTFT.

   \textbf{Note:} \( 45 > 1 + 4 + 3 + 2 + 5 + 7 + 7 + 5 + 2 + 3 + 4 + 1 \), so the phase is uniquely specified here.

[20] 6. Download \texttt{p5.mat}. In Matlab, type \texttt{>>load p5.mat} to get the sampled signal \( X \).

   [5] (a) Listen to \( X \) using \texttt{soundsc(X,24000)}. Describe it. (It’s not the same as before.)
   [5] (b) Plot the spectrum of \( X \) using the Matlab command from problem set #1.
   Compare carefully to the spectrum plot from problem set #1. What did I do?
   [5] (c) Use \texttt{fftshift} somehow and \( Y = \text{real}(\text{ifft}(FY)) \) to unscramble \( X \).
   [5] (d) Use the modulation property of the DTFT to unscramble \( X \) more easily.

   \textbf{Hint:} Shifting discrete-time frequency by \( \pi \) does what in the time domain?

   \textbf{Note:} This is the digital version of \textit{voice scramblers} used in World War II.

   Turn in plots of the spectrum of \( X \) and of its unscrambled version.

“A chicken is just an egg’s way of making another egg.”