ASSIGNED: Jan. 22, 2015. **READ:** Sects. 2.5 & 3.1-3.3 on z-transforms.
DUE DATE: Jan. 29, 2015. **TOPICS:** LTI systems and convolution.

Please box your answers. Show your work. Turn in all Matlab plots and Matlab code.

[30] 1. An ideal digital *differentiator* is implemented as follows:

x(t)→[ANTI–ALIAS]→[A/D]→x[n]→[h[n]]→y[n]→[D/A]→y(t)

where: $y[n]=(x[n+1]-x[n-1])-\frac{1}{2}(x[n+2]-x[n-2])+\frac{1}{3}(x[n+3]-x[n-3])+\ldots$

[5] (a) Is the system LTI? [5] (b) Is it causal? [5] (c) Read off its impulse response $h[n]$.
[5] (d) Prove that this system is *not* BIBO stable. HINT: $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges.
[10] (e) Give a bounded input $x[n]$ that produces an unbounded output $y[n]$. HINT: $h[n]$.

[20] 2. Compute the following convolutions *by hand*. You may check answers using `conv`.

[5] (a) $\{\underline{3},4,5\}*\{\underline{6},7,8\}$. [5] (b) $\{\underline{1},2,-3\}*u[n]$.
[5] (c) $\{\underline{3},4,5\}*(u[n]-u[n-3])$. [5] (d) $\{\underline{1},2,4\}*2\delta[n-2]$.

[20] 3. The following two input-output pairs are observed for a system known to be linear:

$\{\underline{1},2,3\}\rightarrow$ [LINEAR] $\rightarrow\{\underline{1},4,7,6\}$ and $\delta[n]\rightarrow$ [LINEAR] $\rightarrow\{\underline{1},3\}$.

[10] (a) Prove the system is *not* time-invariant. HINT: Prove this by contradiction.
[10] (b) We observe $\{\underline{1},2,3\}\rightarrow\{\underline{1},4,7,6\}$ for a system known to be LTI. Compute h[n].

[10] 4. The AR difference equation $y[n]-y[n-1]-y[n-2]=x[n]$ with input $x[n]=\delta[n]$=impulse generates the sequence of *Fibonacci numbers*. Compute the first 7 Fibonacci numbers:
[5] (a) By recursively computing $y[0],y[1]\ldots y[6]$ directly from the difference equation.
[5] (b) Using `Y=filter(B,A,X);Y(1:7)` with `B=[1];A=[1 -1 -1];X=[1 zeros(1,6)];`. Tape a printout of your Matlab output (only a couple of lines) to your submission. In general, `filter(B,[1],X)` gives the first `length(X)` numbers of `conv(B,X)`.

[20] 5. Download `p2.mat` from the web site. >>`load p2.mat` to get the sampled signal `X`.
[5] (a) Listen to `X` using `soundsc(X,24000)`. Describe it. (It's not the same as before.)
[5] (b) Plot the spectrum of `X` using the Matlab command from problem set #1. How does it differ from the spectrum you computed in problem set #1?
[5] (c) *Filter* `X` using `Y=conv(X,H)`. Listen to `Y` and describe it. Here `H=[0.005 0 -0.042 0 0.29 0.5 0.29 0 -0.042 0 0.005]`.
[5] (d) Plot the spectrum of `Y` using the Matlab command from problem set #1. Explain what the convolution did to the noise that was added to `X`.

"Chance favors the prepared mind"-Louis Pasteur.