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1. **E.** Compute directly. But note periodic $x[n]$ real and even $\rightarrow X_k$ real and even.
This and DC value=8=mean value of $x[n]$ saved you some work: $X_0=(4)(8)=32$.
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2. **B.** $\cos(\frac{\pi}{4}n) = \frac{1}{2}e^{j1\frac{2\pi}{8}n} + \frac{1}{2}e^{-j1\frac{2\pi}{8}n} \rightarrow X_1, X_{8-1}=4$; other $X_k = 0 \rightarrow \{0, 4, 0, 0, 0, 0, 0, 4\}$.
Had to remember that first element of DFT is X_0 =DC component.
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3. **E.** Right side=notch filter with $1 = h[1] = -2 \cos(\omega_o) \rightarrow \omega_o = \frac{2\pi}{3}$.
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4. **E.** Notch filter with $h[1] = -2 \cos(2\pi \frac{375}{1000}) = \sqrt{2}$.
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5. **D.** $y[n] = 8 \cos(\frac{\pi}{2}n) + 3 \cos(\frac{\pi}{2}n - \frac{\pi}{2}) + 4 \cos(\frac{\pi}{2}n - \pi)$.
 $\rightarrow 8 + 3e^{-j\pi/2} + 4e^{j\pi} = 8 - 3j - 4 = 4 - 3j = 5e^{-j37^\circ}$.
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6. **E.** $H(e^{j\omega}) = 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega}$. Plugging in $\omega = \pi/2, \pi$:
 $H(e^{j\pi/2}) = 1 - j - 1 + j = 0$. $H(e^{j\pi}) = 1 - 1 + 1 - 1 = 0$. Output=0!
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7. **B.** $H(e^{j\omega}) = \frac{2e^{-j\omega}}{1+3e^{-j2\omega}} \rightarrow H(z) = \frac{2z^{-1}}{1+3z^{-2}} = \frac{Y(z)}{X(z)} \rightarrow Y(z)(1+3z^{-2}) = X(z)2z^{-1}$.
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8. **A.** Convolve 2 notch filters: $\{1, -1, 1\} * \{1, 1, 1\} = \{1, 0, 1, 0, 1\}$.
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9. **B.** $H(z) = \frac{1-z^{-2}}{1+z^{-2}} \rightarrow H(e^{j\omega}) = \frac{1-e^{-j2\omega}}{1+e^{-j2\omega}} = \frac{e^{j\omega}-e^{-j\omega}}{e^{j\omega}+e^{-j\omega}} = \frac{2j \sin \omega}{2 \cos \omega} = j \tan \omega$.
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10. **B.** Need linear phase. Only a=1 works: $H(e^{j\omega}) = 3 + 2 \cos(\omega) \geq 0$.
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11. **A.** Need minimum phase: all zeros inside unit circle. Only a=1/2 works.
Since product of roots is $a/1$, need $a < 1$. Just check $z^2 + z + \frac{1}{2} = 0$.
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12. **C.** All others periodic with period=2π. (b),(e) (not (d)) $\rightarrow x[n]$ pure imaginary.
Note that $|\sin(\omega/2)|$ =rectified sinusoid (recall circuits?) has period=2π, not 4π.
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13. **E.** $\{3 \cdot 2 + 1 \cdot 1 + 4 \cdot 7, 3 \cdot 7 + 1 \cdot 2 + 4 \cdot 1, 3 \cdot 1 + 1 \cdot 7 + 4 \cdot 2\} = \{35, 27, 18\}$.
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14. **B.** $|H(e^{j\omega})|$ from $H(e^{j\omega}) = \frac{3+4e^{-j\omega}}{1+2e^{-j\omega}} \rightarrow H(z) = \frac{3+4z^{-1}}{1+2z^{-1}} \rightarrow y[n] + 2y[n-1] = 3x[n] + 4x[n-1]$.
Cross-multiply $\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\text{function}(3,4)}{\text{function}(1,2)}$. Imagine if this had been fill-in-the-blank!
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15. **E.** $H(e^{j0}) = 1 + 4 + 3 = 8$. $H(e^{j\pi/2}) = 1 - 4j + 3j = 1 - j = \sqrt{2}e^{-j\pi/4}$.
 $H(e^{j\pi}) = 1 - 4 - 3 = -6$. $y[n] = 8(1) + \sqrt{2}(2) \cos(\frac{\pi}{2}n - \frac{\pi}{4}) - 6(3) \cos(\pi n)$.
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16. $x[n]$ even and periodic with period=8 \rightarrow can expand $x[n]$ in a DTFS (Fourier series).
 $x[n] = a_0 + a_1 \cos(\frac{2\pi}{8}n) + a_2 \cos(\frac{4\pi}{8}n) + a_3 \cos(\frac{6\pi}{8}n) + a_4 \cos(\pi n)$.
 $H(z) = 1 - z^{-2} + z^{-4} - z^{-6} = \frac{z^6 - z^4 + z^2 - 1}{z^6} = \frac{z^8 - 1}{(z^2 + 1)z^6} \rightarrow$ zeros at $\pm j$ cancelled by poles \rightarrow
 $\omega = 0, \pm \frac{\pi}{4}, \pm \frac{3\pi}{4}, \pi$ all rejected, allowing through only $\omega = \pm \frac{\pi}{2} \rightarrow y[n] = A \cos(\frac{\pi}{2}n)$.
Half credit if you thought this filter rejected *all* frequencies, or passed only $\cos(\pi n)$.
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17. Poles at $0.99e^{\pm j\pi/3} \rightarrow$ sharp peaks at $\omega = \pm \pi/3$. Zeros at $e^{\pm j2\pi/3} \rightarrow$ dips.
Full credit for the correct shape (zeros and peaks at right places).

SCORES	95	90	85	80	75	70	65	60	55	< 55	Total	Mean	Median
#ugrad	4	10	8	10	7	9	10	3	4	1	66	75.8	75 - 80
#grad	1	3	1	1	0	0	0	0	0	1	7	82.9	90