1. C. Discrete time! \( 0.075 = \frac{75}{1000} = 3/40 \) lowest terms \( \rightarrow \) period \( = \) denominator \( = \) 40.

2. A. \( \sin(32\pi \frac{n}{40}) + \sin(48\pi \frac{n}{40}) = \sin(0.8\pi n) + \sin(1.2\pi n) = 0. \)
   Replace “sin” with “cos” throughout and the answer is (e).


4. C. \( Y(z)[1 + 2z^{-1} + 3z^{-2}] = X(z)[4 + 5z^{-1} + 6z^{-2}] \rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{4z^2 + 5z + 6}{z^2 + 2z + 3}. \)

5. E. \( H(z) = Z\{h[n]\} = \frac{z}{z^2 + 2} + \frac{z^2}{z^2 + 2} = \frac{2z^2 - 3z}{z^2 + 2z + 3}. \)

6. E. \( \frac{z^2}{z^2 + 2} = \frac{z}{z^2 - 2} \)

7. E. \( \frac{z}{z^2 - 2} \)

8. A. \{ 1, -3, 2 \} \# u(n) = u(n) - 3u(n-1) + 2u(n-2) = \{ 1, -2 \} \cdot (1 - 3z^{-1} + 2z^{-2}) \rightarrow h[n] = \frac{2z}{z^2}. \)

9. D. \( H(z) = \frac{2z - 2z^{-1}}{z^2 - 2} = \frac{2z - 2z^{-1}}{2z^2 - 2} = 1 - 2z^{-1} \rightarrow h[n] = \{ 1, -2 \}. \) Same as #8!

10. A. 1-sided: \( Y^+(z) - 2z^{-1}Y^+(z) + 2y(-1) = 0 \rightarrow Y^+(z) = \frac{2z}{z^2} \rightarrow y[n] = 2(2)^{n}u[n]. \)

11. B. Poles \{ 2, 3 \} both outside unit circle \( \rightarrow \) unstable. I should have specified “causal.”

12. C. \( H(z) = \frac{Y(z)}{X(z)} = \frac{z^2 - 7z + 6}{z^2 - 5z + 6}. \) Cross-multiply: \( Y(z)(z^2 - 5z + 6) = X(z)(z^2 - 7z + 6). \) Z \( \rightarrow \) 1.

13. A. \( Y(z) = H(z)X(z) = \frac{z^2 - 7z + 6}{z^2 - 5z + 6} \rightarrow y[n] = \{ 1, -7, 6 \}. \)

14. A. \( y[n] = h[n] \ast \sum_{n=0}^{\infty} h(n) = 7H(1) = 0. \) OK, I shouldn’t have asked this one.

15. D. \( \frac{H(z)}{z} = \frac{(z - 1)(z - 6)}{z(z - 2)(z - 1)} = \frac{1}{z} + \frac{2}{z - 1} - \frac{2}{z - 2} \rightarrow H(z) = 1 + 2z - 2z^{-1}. \)

16. (i) \( (0.5)^{n-1}u[n - 1] - 3^n - 1u[n - 1] \) \{ \( z : |z| > 3 \) \} causal unstable

   (ii) \( (0.5)^{n-1}u[n - 1] + 3^n - 1u[u - n] \) \{ \( z : 0.5 < |z| < 3 \) \} noncausal stable

   (iii) \( 3^n - 1u[-n] - (0.5)^{n-1}u[-n] \) \{ \( z : |z| < 0.5 \) \} anticausal unstable

Scores: # undergrad # grad problem grading comments

| 100 | 5 | 1 | 1 | Discrete time! Aargh... |
| 95 - 99 | 12 | 3 | 7 | (d) Must replace n \( \rightarrow \) n - 1 |
| 90 - 94 | 14 | 2 | 12 | Trouble with this? Aargh... |
| 85 - 89 | 12 | 0 | 14 | Much harder than I thought |
| 80 - 84 | 10 | 0 | 16 | -3 if thought \( \frac{z}{z - 0.5} - \frac{z}{z - 3} \) |
| 75 - 79 | 4 | 0 | 16 | I admit it: I forgot z's |
| 70 - 74 | 5 | 0 | 16 | -2 if u[-n - 2] not u[-n] |
| 65 - 69 | 3 | 1 | 16 | u[-(n - 1) - 1] = u[-n]. Delay : |
| 60 - 65 | 3 | 1 | 16 | Replace n \( \rightarrow \) n - 1 everywhere |
| TOTAL | 67 | 7 | 15 | Put answer in different form |

Mean: 85.5 90.3
Pretty good! Was it too easy?