Given: A circuit with several inductors and capacitors and different sources.
Also: Non-zero initial conditions: capacitors charged and inductors “juiced.”
Goal: Compute any circuit voltage or current as a function of time \( t \) for \( t > 0 \).
Soln: Solve constant-coefficient differential equation with initial conditions.
But: Lots of algebra, even using Laplace transform (have to obtain diff. eqn.)
But: Is there easier way, like phasors but for any source and initial conds.?

Idea: Take Laplace transform of entire circuit. Using \( \mathcal{L}\{ \frac{dx}{dt} \} = sX(s) - x(0^+) \),

<table>
<thead>
<tr>
<th>Device Name</th>
<th>Resistor</th>
<th>Inductor</th>
<th>Capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Its Formula</td>
<td>( v = Ri )</td>
<td>( v(t) = L \frac{di}{dt} )</td>
<td>( i(t) = C \frac{dv}{dt} )</td>
</tr>
<tr>
<td>After Lxfm</td>
<td>( V = RI )</td>
<td>( V = L(sI - i(0)) )</td>
<td>( I = C(sV - v(0)) )</td>
</tr>
<tr>
<td>Impedance</td>
<td>( R )</td>
<td>( sL )</td>
<td>( \frac{1}{sC} )</td>
</tr>
<tr>
<td>Device Model</td>
<td>( -R )</td>
<td>( -sL )</td>
<td>( -\frac{1}{sC} )</td>
</tr>
</tbody>
</table>

1. **Sources** have their values replaced with their Laplace transforms.
   - Note that a **constant source** \( A \) volts or amps is replaced with \( \frac{A}{s} \).
   - Dependent sources now depend on some circuit variable \( V(s) \) or \( I(s) \).
2. **Capacitor**: \( I = C(sV - v(0)) \rightarrow V = \frac{1}{sC}I + \frac{v(0)}{s} \) (note the + sign).
   - Can also use the **Norton equivalent**: \( \left( \frac{1}{sC} \right) || \left( \text{current source} = Cv(0) \right) \).
3. Note **sign difference**: Voltage sources for \( L \) and \( C \) initial conditions.
4. Now straightforward circuit analysis. Like phasors except \( s \), not \( j\omega \).
5. After compute desired \( V(s) = \mathcal{L}\{v(t)\} \) or \( I(s) = \mathcal{L}\{i(t)\} \), compute \( \mathcal{L}^{-1} \).
   - Note \( V(s) \) or \( I(s) \) will be a rational function, so use partial fractions.

**EXAMPLE: RC CIRCUIT DRIVEN BY STEP FUNCTION**

Given: Series RC circuit driven by step voltage source \( A \cdot 1(t) \)
Goal: Compute capacitor voltage
With: Initial capacitor voltage=\( B \)

\[ \text{Soln: KVL} \rightarrow \frac{A}{s} - I(s)R - I(s)\frac{1}{sC} - \frac{B}{s} = 0 \rightarrow I(s) = \frac{\frac{A}{s} - \frac{B}{s}}{\frac{1}{sC} + \frac{1}{sC}} = \frac{(A-B)/R}{s+1/(RC)} \]

Then: \( V(s) = \frac{B}{s} + \frac{1}{sC}I(s) = \frac{B}{s} + \frac{(A-B)/(RC)}{s(s+1/(RC))} \). Compute partial fractions:

Then: \( V(s) = \frac{A}{s} + \frac{B-A}{s+1/(RC)} \rightarrow v(t) = \frac{A}{s} + (B-A)e^{-t/(RC)} \) for \( t > 0 \).

**Also**: Can rewrite this as \( v(t) = A(1 - e^{-t/(RC)}) + Be^{-t/(RC)} \) for \( t > 0 \).
**REDO PREVIOUS 2\textsuperscript{nd}-ORDER CIRCUIT EXAMPLE:**

**Given:** Circuit shown at right:

**Initially:** \( i(0) = 0 \) and \( v(0) = -1 \)

**Then:** Throw switch at \( t = 0 \).

**Goal:** Compute \( i(t) \) for \( t > 0 \)

KVL: \( \frac{2s}{s^2+9} - 4I - s1I + 0 - \frac{1}{s^3}I = 0 \rightarrow I = \frac{\frac{2s}{s^2+9} + \frac{1}{s}}{4 + s + \frac{3}{s}} \).

Solve: \( I = \frac{3s^2+9}{(s^2+9)(s^2+4s+3)} = 0.223e^{j27^\circ} + 0.223e^{-j\cdot27^\circ} + \frac{0.6}{s+1} + \frac{-1}{s+3} \) PARTIAL FRACTION

\( \mathcal{L}^{-1} \): \( i(t) = 0.223e^{-j3t+j\cdot27^\circ} + 0.223e^{j3t-j\cdot27^\circ} + 0.6e^{-t} - e^{-3t} \) for \( t > 0 \)

**Then:** Simplifies to: \( i(t) = 0.447\cos(3t - 27^\circ) + 0.6e^{-t} - e^{-3t} \) for \( t > 0 \).

1. This agrees with previous handout result using differential equations.
2. Note characteristic equation appears in denominator of \( I(s) \) expression.
3. Partial fraction shows where transient and steady-state responses arise.

**Given:** The circuit shown at right:

**Initially:** Switch closed for long time.

**Then:** Switch is opened at \( t = 0 \).

**Goal:** Compute \( v_C(t) \) for \( t > 0 \).

\( t = 0^- \): \( C \rightarrow \) open and \( L \rightarrow \) short \( \rightarrow i_L(0^-) = \frac{10}{50} = 0.2 \) and \( v_C(0^-) = 0 \).

\( t = 0^+ \): We don’t need rest of the messy computation from previous handout!

**Now:** With the switch open, this is now just a voltage divider for \( V_C(s) \):

\[
V_C(s) = \frac{\frac{1}{0.02s}}{1/((0.02s)+1)+5s}(-5(0.2)) = \frac{-10}{s^2+2s+10} = \frac{(-3.33)(3)}{(s+1)^2+(3)^2} \textrm{ so that:}
\]

\( \mathcal{L}^{-1} \): \( v_c(t) = -3.33e^{-t}\sin(3t) \) for \( t > 0 \). Agrees with the previous handout.

1. We don’t need to go through previous mess of finding \( \frac{dv_C}{dt}(0^+) \) (ugh).
2. We can compute inverse Laplace transform without partial fractions.
3. Note characteristic equation appears in denominator of \( I(s) \) expression.

**Given:** Series RLC circuit (resistor+inductor+capacitor connected together).

**Initial:** Capacitor is charged up to \( v(0) \) and inductor is “juiced up” to \( i(0) \).

**t=0:** Close or throw the switch at \( t = 0 \). Recall \( i(t) \) and \( v(t) \) don’t jump.

**Goal:** Compute current through inductor (and everything else) \( i(t) \) for \( t > 0 \).

KVL: \( LI(0) - sLI - RI - \frac{1}{sC}I - \frac{v_c(0)}{s} = 0 \rightarrow I(s) = \frac{Li(0)-v(0)/s}{sL+R+1/(Cs)} \). Easy!

**Soln:** \( i(t) = \mathcal{L}^{-1} \) of \( I(s) = \frac{si(0)-v(0)/L}{s^2+\frac{R}{L}s+\frac{1}{LC}} \). Note the characteristic equation.