LINE SPECTRUM OF SIGNAL=SUM OF SINUSOIDS

Given: $x(t) = A_0 + A_1 \cos(\omega_1 t + \theta_1) + \ldots + A_N \cos(\omega_N t + \theta_N)$.
Sum of N sinusoids **at** N **different frequencies**.

Goal: Represent amplitudes and phases as scaled impulses vs. frequency.

Using $Re[z] = \frac{1}{2}(z + z^*)$ with $z = X_k e^{j\omega_k t}$, we have:

- $x(t) = A_0 + \frac{1}{2}\sum_{k=1}^{N}(X_k e^{j\omega_k t} + X_k^* e^{-j\omega_k t})$ where $X_k = A_k e^{j\theta_k}$.

1. Can represent a sinusoid at frequency $\omega_k = 2\pi f_k$ by *two complex exponentials* at frequencies $\pm\omega_k$.
2. Note need negative frequencies; watch amplitude factor of two.
3. Note component at negative frequency $-\omega_k$ is complex conjugate of its counterpart component at positive frequency ω_k.
4. Plot: impulses=lines in frequencies; hence *line spectrum*.

EX: $x(t) = 10 + 14\cos(2t - 1) + 8\cos(4t + 3)$ has 5 complex components:
(10 at $\omega = 0$); ($7e^{\mp j}$ at $\omega = \pm 2$); ($4e^{\pm j3}$ at $\omega = \pm 4$) (see plot overleaf).

BEAT SIGNALS (SOUND LIKE THEY ARE BEATING)

What: Identity $\cos(\omega_1 t + \theta_1) + \cos(\omega_2 t + \theta_2) = 2\cos(\omega_c t + \theta_c)\cos(\omega_\Delta t + \theta_\Delta)$.

where: $\omega_c = \frac{1}{2}(\omega_1 + \omega_2)$ and $\omega_\Delta = \frac{1}{2}(\omega_1 - \omega_2)$; θ_c, θ_Δ are defined similary.

Why? Apply cosine addition formula and $\omega_1 = \omega_c + \omega_\Delta$ and $\omega_2 = \omega_c - \omega_\Delta$.

Easier: Note that $2\cos(\omega_c t + \theta_c) 2\cos(\omega_\Delta t + \theta_\Delta)$ can be written as product
$[e^{j(\omega_c t + \theta_c)} + e^{-j(\omega_c t + \theta_c)}][e^{j(\omega_\Delta t + \theta_\Delta)} + e^{-j(\omega_\Delta t + \theta_\Delta)}]$.
Multiply this out$\rightarrow$4 terms$\rightarrow$ $2\cos(\omega_1 t + \theta_1) + 2\cos(\omega_2 t + \theta_2)$ (try it!).

Sounds like: $x(t)$ sounds like a single tone at frequency $\omega_c \approx \omega_1, \omega_2$
whose *amplitude varies slowly with time* at frequency ω_Δ.

EX #1: **Tuning a piano**. We wish to tune a piano key to 440 Hz.

Given: A tuning fork tuned to A above middle C, which is 440 Hz.

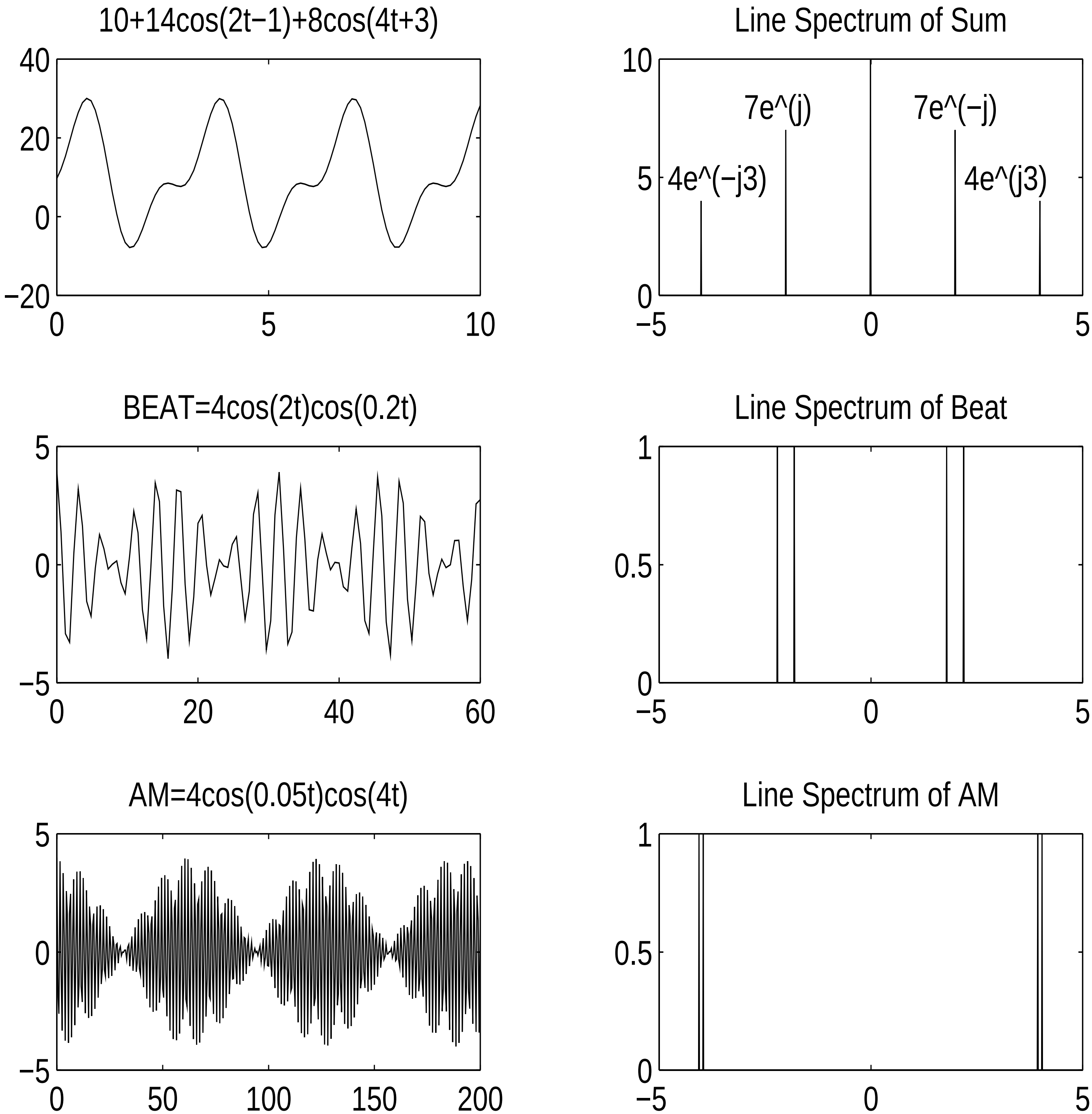
Given: Piano mistuned to 441 Hz. Want to change this to 440 Hz.

Sol'n: Strike tuning fork and piano key simultaneously and listen.
$\rightarrow$ Hear $\cos(2\pi 440 t) + \cos(2\pi 441 t) = 2\cos(2\pi 0.5 t)\cos(2\pi 440.5 t)$

Sounds like: A single tone at 440.5 Hz whose amplitude varies at 0.5 Hz.
Actually 1 Hz since amplitude is non-negative. Period=1 second.

Sol'n: Adjust piano key until amplitude variation ceases (Period$\rightarrow\infty$).

EX #2: **AM radio**. WJR (760 kHz) transmits a tone (EBS?) at 5 kHz.
$x(t) = 4\cos(2\pi(760,000)t)\cos(2\pi(5000)t)$
$= 2\cos(2\pi(755,000)t) + 2\cos(2\pi(765,000)t)$
$= e^{j2\pi(765,000)t} + e^{j2\pi(755,000)t} + e^{-j2\pi(765,000)t} + e^{-j2\pi(755,000)t}$.
Line spectrum: 4 components in two closely-spaced pairs.


10+14cos(2t−1)+8cos(4t+3)
40
20
0
−20
0
5
10
Line Spectrum of Sum
10
5
0
−5
0
5
7e^(j)
7e^(−j)
4e^(−j3)
4e^(j3)
BEAT=4cos(2t)cos(0.2t)
5
0
−5
0
20
40
60
Line Spectrum of Beat
1
0.5
0
−5
0
5
AM=4cos(0.05t)cos(4t)
5
0
−5
0
50
100
150
200
Line Spectrum of AM
1
0.5
0
−5
0
5