absolutely impulse response: $\sum |h[n]|$ is finite. EX: $\sum_{n=0}^{\infty} (\frac{3}{4})^n = \frac{1}{1 - \frac{3}{4}} = 4$ but $\sum_{n=1}^{\infty} \frac{1}{n} \to \infty$.

summable Necessary and sufficient for BIBO stability of an LTI system. Also see poles.

affine $y[n] = ax[n] + b$ for constants $a$ and $b \neq 0$. This is a linear-plus-constant function.

system Affine systems are not linear systems, although they do look like linear systems.

aliasing EX: $x(t) = \cos(2\pi ft)$ sampled at 10 Hz: $t = \frac{n}{10} \to x[n] = \cos(1.2\pi n) = \cos(0.8\pi n)$. Ideal interpolation (for a sinusoid) $n = 10t \to x(t) = \cos(2\pi ft)$. 6 Hz aliased to 4. Can avoid aliasing by sampling faster than Nyquist rate, or use an antialias filter.

amplitude of a sinusoid: $|A|$ in sinusoidal signal $A \cos(\omega t + \theta)$. Amplitude is always $\geq 0$.

antialias Analog lowpass filter that ensures that the sampling rate exceeds Nyquist rate, filter ensuring that aliasing does not occur during interpolation of the sampled signal.

argument of a complex number: another name for phase. Matlab: angle(1+j)=0.7854=π/4.

ARMA Auto-Regressive Moving-Average difference equation. #$y[n] > 1$ and #$x[n] > 1$.

average Continuous time: Average power=$MS(x) = \frac{1}{T} \int_0^T |x(t)|^2 dt$ for $x(t)$ period=T.

power Discrete time: Average power=$MS(x) = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$ for $x[n]$ period=N.

band-pass Filter with frequency response $H(e^{j\omega})$ small at DC $\omega = 0$ and small at $\omega = \pi$, but large at some intermediate band $a < \omega < b$. BPFs reduce high-frequency noise.

bandwidth Maximum frequency in a cont.-time signal $x(t)$. Sometimes defined as twice this.

beat EX: cos(99t) + cos(101t) = 2 cos(1t) cos(100t) like varying amplitude cos(100t).

signal This beating sound can be used to tune a piano; also models a tone on AM radio.

BIBO Bounded-Input-Bounded-Output system: $x[n] \to |BIBO| \to y[n]$. Means: if $|x[n]| \leq M$ for some constant $M$, then $|y[n]| \leq N$ for some constant $N$. An LTI system is BIBO stable iff all of its poles are inside the unit circle.

cascade connection of systems: $x[n] \to |g[n]| \to |h[n]| \to y[n]$. Overall impulse response: $y[n] = h[n] * (g[n] * x[n]) = (h[n] * g[n]) * x[n] \to \text{convolve their impulse responses}$.

causal: signal: $x[n] = 0$ for $n < 0$; causal system has a causal impulse response $h[n]$.

comb $y[n] = x[n] + x[n-1] + \ldots + x[n-M+1]$ eliminates all $A_k \cos(\frac{2\pi}{M} kn + \theta_k)$ in $x[n]$.

filter transfer function$=H(z) = (z^{-1} e^{j\frac{2\pi}{M} k}) \ldots (z^{-1} e^{j\frac{2\pi}{M} (M-1)}) / z^{M-1}$ has zeros at $e^{j\frac{2\pi}{M} k}$, which are the $M^{th}$ roots of unity. Eliminates all harmonics of periodic function.

complex of complex number $z = x + jy$ is $z^* = x - jy$; of $z = Me^{j\theta}$ is $z^* = Me^{-j\theta}$.

conjugate Properties: $|z|^2 = zz^*$; $Re[z] = \frac{1}{2}(z + z^*)$; $Im[z] = \frac{1}{2j}(z - z^*)$; $e^{j2\pi z} = \frac{z}{z^*}$.

complex Continuous time: $x(t) = e^{j\omega t}$. Discrete time: $x[n] = e^{j\omega n}$ where $\omega =$frequency.

exponential Fourier series are expansions of periodic signals in terms of these functions.
**complex plane** You can visualize $z = 3 + j4$ at Cartesian coordinates $(3, 4)$ in the complex plane.

**conjugate** If $x(t)$ is real-valued, then $x_{-k} = x_k^*$ in its Fourier series expansion, where $x_k^*$ is complex conjugate of $x_k$. **DFT:** If $x[n]$ is real-valued, then $X_{N-k} = X_k^*$

**convolution** $y[n] = h[n] * x[n] = \sum_{i=0}^{n} h[i] x[n-i]$. $x[n] \rightarrow \text{LTI} \rightarrow y[n] = h[n] * x[n]$ where $h[n]=$ impulse response of LTI system.

**correlation** Continuous time: $C(x, y) = \int x(t)y(t)^*dt$. Discrete time: $C(x, y) = \sum x[n]y[n]^*$.

**correlation coefficient** $C_N(x, y) = \frac{C(x, y)}{\sqrt{C(x,x)C(y,y)}}$ where $C(x, y)=$correlation and $C(x, x) = E(x) =$energy. $|C_N(x, y)| \leq 1$ by Cauchy-Schwarz, so $C_N(x, y) =$ (the similarity of $x[n]$ and $y[n]$).

**delay** $x[n]$ delayed by $D > 0$ is $x[n - D]$: $D$ seconds later; shift $x[n]$ graph right by $D > 0$.

**DFT** N-point DFT of $\{x[n]\}$ is $X_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$ for $k = 0, 1 \ldots N-1$. Use **fft**. Discrete Fourier Transform computes coefficients $X_k$ of the discrete-time Fourier series $x[n] = X_0 + X_1 e^{j\frac{2\pi}{N}n} + X_2 e^{j\frac{4\pi}{N}n} + \ldots + X_{N-1} e^{j\frac{2\pi(N-1)}{N}n}$ where $x[n]$ has period=N.

**difference** $y[n] = a_1y[n-1] + \ldots + a_Ny[n-N] = b_0x[n] + b_1x[n-1] + \ldots + b_Mx[n-M]$. **equation** ARMA; describes an LTI system; this is how it is actually implemented on a chip.

**duration** Continuous time: If signal support=[a, b] → duration=(b - a)=how long it lasts. Discrete time: If signal support=[a, b] → duration=(b - a + 1). Note the “+1”.

**energy** Continuous: $E(x) = \int_a^b |x(t)|^2dt$. Discrete: $E(x) = \sum_{n=a}^{b} |x[n]|^2$ if support=[a, b].

**Euler’s formula:** $e^{j\theta} = \cos \theta + j\sin \theta$. **Polar⇒rectangular:** $Me^{j\theta} = M\cos \theta + jM\sin \theta$.

**even** $x(t) = x(-t)$: $x(t)$ is symmetric about the vertical axis $t = 0$. **EX:** $\cos(t), t^2, t^4$.

**function** A periodic even function has **Fourier series** having $b_k = 0$ and $x_k$ real numbers.

**fft** Matlab command that computes DFT. Use: **fft**(x,N)/N. See also **fftshift**.

**fftshift** Matlab command that swaps first and second halves of a vector. Used with **fft**. Displays the line spectrum with negative frequencies to left of positive ones.

**filter** LTI system with frequency response performing some task, e.g., noise reduction.

**FIR** Finite Impulse Response system: its impulse response $h[n]$ has finite duration. FIR systems are: also MA systems; always BIBO stable; all poles are at origin.

**Fourier** $x(t) = x_0 + x_1 e^{j\frac{2\pi}{T}t} + x_2 e^{j\frac{4\pi}{T}t} + \ldots + x_N e^{j\frac{2\pi N}{T}t}$ series where $x_k = \frac{1}{T} \int_0^T x(t)e^{-j\frac{2\pi}{N}kt} dt$ for integers $k$ and $x(t)$ is real-valued with period=T.
Fourier $x(t) = a_0 + a_1 \cos(\frac{2\pi}{T} t) + a_2 \cos(\frac{4\pi}{T} t) + \ldots + b_1 \sin(\frac{2\pi}{T} t) + b_2 \cos(\frac{4\pi}{T} t) + \ldots$ where

series $a_k = \frac{1}{T} \int_0^T x(t) \cos(\frac{2\pi}{T} k t) \, dt$ and $b_k = \frac{1}{T} \int_0^T x(t) \sin(\frac{2\pi}{T} k t) \, dt$ and $x(t)$ has period $= T$.

(trig form) BUT: note $a_0 = \frac{1}{T} \int_0^T x(t) \, dt = M(x)=\text{DC term}$ is a special case: note $\frac{1}{T}$, not $\frac{2}{T}$.

frequency of a sinusoid: $\omega$ in signal $A \cos(\omega t + \theta)$. Units of $\omega$: $\text{RAD/SEC}$. Units of $f = \frac{\omega}{2\pi}$: Hertz.

frequency $H(e^{j\omega}) = H(z)|_{z=e^{j\omega}}$ where $H(z)$ is transfer function. Sometimes write: $H(\omega)$.

response $\cos(\omega_0 n) \rightarrow \boxed{\text{LTI}} \rightarrow A \cos(\omega_0 n + \theta)$ where $H(e^{j\omega_0}) = Ae^{j\theta}$. $A$=gain of system.

$x[n] = \sum X_k e^{j\frac{2\pi}{T}kn} \rightarrow \boxed{\text{LTI}} \rightarrow y[n] = \sum Y_k e^{j\frac{2\pi}{T}kn}$ where $Y_k = H(e^{j\frac{2\pi}{T}k})X_k$.

fundamental Sinusoid $c_1 \cos(\frac{2\pi}{T} t + \theta_1)$ at frequency $\frac{1}{T}$ Hertz in Fourier series of periodic $x(t)$.

gain $\text{Gain}=|H(e^{j\omega})|=$magnitude of frequency response. Describes filtering effect.

harmonic Sinusoid $c_k \cos(\frac{2\pi}{T} kt + \theta_k)$ at frequency $\frac{k}{T}$ Hz in Fourier series of periodic $x(t)$.

high-pass Filter with frequency response $H(e^{j\omega})$ small at DC $\omega = 0$ and large at $\omega = \pi$.

filter (HPF) HPFs enhance edges in images and signals, but also amplify high-frequency noise.

histogram Plot of #times a signal value lies in a given range vs. range of signal values (bins).

Can be used to compute good approximations to mean, mean square, rms, etc.

IIR Infinite Impulse Response system: impulse response $h[n]$ has infinite duration.

imaginary of a complex number: $Im[x+jy] = y$; $Im[Me^{j\theta}]=M \sin \theta$. Matlab: imag(3+4j)=4.

part Note that the imaginary part of $3+j4$ is 4, NOT j4! A VERY common mistake!

impulse Discrete time: $\delta[n]=0$ unless $n=0$; $\delta[0]=1$. Continuous time: wait for EECS 306.

impulse $h[n]$: Response to an impulse: $\delta[n] \rightarrow \boxed{\text{LTI}} \rightarrow h[n]$. $y[n]=$$b_0x[n]+\ldots+b_Mx[n-M]$,

response $x[n] \rightarrow \boxed{\text{LTI}} \rightarrow y[n] = h[n] * x[n]$; see convolution. then $h[n] = \{b_0,b_1\ldots b_M\}$.

inter- A signal nature (60 Hz) or humans (jamming) adds to a desired signal that obscures the desired signal. Unlike added noise, interference is usually known approximately.

ference

interpolation The act of reconstructing $x(t)$ from its samples $x[n]=x(t=n\Delta)$. Also: D-to-A.

Exact interpolation of discrete Fourier series: set $n = t/\Delta$ in all the sinusoids.

inverse Get $x[n]$ from its z-xform $X(z) = \mathcal{Z}\{x[n]\}$. EX: $X(z) = \frac{z}{z-3} \rightarrow x[n] = 3^n u[n]$.

z-xform Do this by inspection or partial fraction expansion of rational function $X(z)$.

line Plot of the $x_k$ in Fourier series of a signal vs. frequency $\omega$. Also for $X_k$ in DFT.

spectrum $x_k$ is depicted in plot by a vertical line of height $|x_k|$ at $\omega = \frac{2\pi}{T}k$ where $T=\text{period}$.
linear combination Lin. system: Response to linear combination is linear combination of responses.

linear This means that if \( x_1[n] \rightarrow \text{LINEAR} \rightarrow y_1[n] \) and \( x_2[n] \rightarrow \text{LINEAR} \rightarrow y_2[n] \)
system then \( (ax_1[n] + bx_2[n]) \rightarrow \text{LINEAR} \rightarrow (ay_1[n] + by_2[n]) \) for any constants \( a \) and \( b \).

Often works: if doubling input \( x[n] \) doubles output \( y[n] \), system is likely to be linear.

low-pass Filter with frequency response \( H(e^{j\omega}) \) large at DC \( \omega = 0 \) and small at \( \omega = \pi \).

filter (LPF) LPFs smooth edges in images and signals, but they do reduce high-frequency noise.

LTI system is both Linear and Time-Invariant. So what? See Impulse Response and Frequency Response.

MA Moving-Average difference equation: \( y[n] = b_0 x[n] + b_1 x[n-1] + \ldots + b_M x[n-M] \).

MA systems are: also FIR systems; always BIBO stable; all poles are at origin.

magnitude of a complex number: \(|Me^{j\theta}| = M; \mid x+jy\mid = \sqrt{x^2+y^2} \). Matlab: abs(3+4j)=5.

Matlab Computer program used universally in communications, control, and signal processing.
Matlab has been known to drive people crazy (look at what happened to me :)).

mean Cont.: \( M(x) = \frac{1}{T} \int_0^T x(t)dt \) for \( x(t) \) period=\( T \). \( M(x) = x_0 = a_0 \) in Fourier series.
Disc.: \( M(x) = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \) for \( x[n] \) period=\( N \). \( M(x) = X_0 \) in the DFT of \( x[n] \).

mean Cont.: \( MS(x) = \frac{1}{T} \int_0^T |x(t)|^2dt = M(x^2) \neq (M(x))^2 \) for \( x(t) \) periodic with period=\( T \).
square Disc.: \( MS(x) = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \) for \( x[n] \) period=\( N \). Mean square=average power.

mean Fourier series Periodic \( x(t) = x_0 + x_1 e^{j\frac{2\pi}{T}t} + \ldots + x_M e^{j\frac{2\pi}{T}Mt} + \ldots \) (infinite series).
square \( \hat{x}(t) = x_0 + x_1 e^{j\frac{2\pi}{T}t} + \ldots + x_M e^{j\frac{2\pi}{T}Mt} + \ldots \) (finite series).
error MSE=MS\( (x(t) - \hat{x}(t)) \) is the mean square error in approximating \( x(t) \) with \( \hat{x}(t) \).

noise An unknown signal that nature adds to a desired signal that obscures desired signal. A lowpass filter often helps reduce noise, while (mostly) keeping the desired signal.

notch \( y[n] = x[n] - 2\cos(\omega_n) x[n-1] + x[n-2] \) eliminates \( A\cos(\omega_n + \theta) \) component in \( x[n] \).
filter transfer function\( = (z-e^{j\omega_n})(z-e^{-j\omega_n})/z^2 \) has: zeros at \( e^{\pm j\omega_n} \); poles at origin.

Nyquist Sampling a continuous-time signal \( x(t) \) at twice its bandwidth (minimum rate).
sampling The minimum sampling rate for which \( x(t) \) can be interpolated from its samples.

odd \( x(t) = -x(-t) \): \( x(t) \) is antisymmetric about the vertical axis \( t = 0 \). EX: \( \sin(t), t, t^3 \).
function A periodic odd function has Fourier series having \( a_k = 0 \) and \( x_k \) pure imaginary.

order of this MA (also FIR) system is \( M: y[n] = b_0 x[n] + b_1 x[n-1] + \ldots + b_M x[n-M] \).

origin of the complex plane is at Cartesian coordinates \( (0,0) \). An elegant name for \( 0+0j \).

orthogonal signals \( x(t) \) and \( y(t) \) have correlation \( C(x,y) = 0 \); \( MS(x+y) = MS(x) + MS(y) \).
complex exponentials and sinusoids that are harmonics yields Fourier series.
Parseval’s theorem states that the average power of a discrete-time signal can be computed in either the time or Fourier domains.

Partial fraction expansion of a rational function $H(z) = \frac{N(z)}{D(z)} = \frac{A_1}{z-p_1} + \ldots + \frac{A_N}{z-p_N}$. Need: degree $D(z) >$ degree $N(z)$. The residues of unity are the poles of the transfer function $H(z)$; $\{A_n\}$ are its residues.

Phase of a complex number: $\angle(Me^{j\theta}) = \theta$; $\angle(x+Jy) = \tan^{-1}(\frac{y}{x})$. Use angle $1+J = \pi/4$.

Phase of a sinusoid: $\theta$ in the sinusoid $A\cos(\omega t + \theta)$. Add $\pi$ if $M < 0$, $A < 0$ or $x < 0$ above.

Poles of a rational-function transfer-function $H(z) = \frac{N(z)}{D(z)}$ are the roots of $D(z) = 0$.

EX: $H(z) = \frac{z^2-3z+2}{z^2-7z+12} \rightarrow z^2-7z+12 = 0 \rightarrow \{\text{poles}\} = \{3, 4\}$. roots([1 -7 12])=[3,4].

An LTI system is BIBO stable if and only if all poles are inside the unit circle.

Pole-zero plot of poles using X’s and zeros using O’s of LTI system in the complex plane.

Quantization of $x[n]$ represents each value of $x[n]$ with a finite #bits=N, so $x[n]$ can take $2^N$ values. Produces slight roundoff error, which is neglected in EECS 206 since N=16 or 32.

Rational function_transfer_function in EECS 206 are rational functions with finite #poles & zeros.

Residues The constants $\{A_n\}$ in the partial fraction expansion of a rational function.

Real part of a complex number: $Re[x+jy] = x$; $Re[Me^{j\theta}] = M \cos \theta$. Matlab: real(3+4j)=3.

Rectangular of complex number: $x+Jy$. Coordinates $(x, y)$ in complex plane. cf. polar form.

RMS $\sqrt{\int_0^T |x(t)|^2 dt}$ of $x(t)$, see Parseval’s theorem $\int_0^T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |x_k|^2 = a_0^2 + \frac{1}{2} \sum_{k=1}^{\infty} (a_k^2 + b_k^2)$ (see Fourier series).

EX: $\text{rms}[A \cos(\omega t + \theta)] = \frac{A}{\sqrt{2}}$ if $\omega \neq 0$.

M roots The $M$ solutions to $z^M = 1$, which are $z = e^{\frac{2\pi}{M}k}$ for $k = 1 \ldots M$. Unity means “1.”

Of unity The zeros of a comb filter are the $M^{th}$ roots of unity, excluding the root $z = 1$. Produces slight roundoff error, which is neglected in EECS 206 since N=16 or 32.
**Correlation** between a signal and **delayed** versions of another signal, regarded as a function of the delay: \( RC[n] = C(x[i], y[i-n]) \). Used for time delay estimation.

**sampling** Act of constructing \( x[n] = x(t = n\Delta) \) from values of \( x(t) \) at integer multiples of \( \Delta \).

**sawtooth** Periodic \( x(t) = at \) for \( 0 < t < b \) and \( x(t) = 0 \) for \( b < t < c \) (like chain of triangles).

**sinc** \( \text{sinc}(x) = \frac{\sin \pi x}{\pi x} \). Used in ideal interpolator, and also in \( h[n] \) for ideal lowpass filter.

**sinusoid** A function of time of the form \( x(t) = A \cos(\omega t + \theta) = (\frac{1}{2}e^{j\theta})e^{j\omega t} + (\frac{1}{2}e^{-j\theta})e^{-j\omega t} \).

**spectrum** See line spectrum. In EECS 206, all spectra are line spectra. Not so in EECS 306.

**square** Periodic \( x(t) = a \) for \( c < t < d \) and \( x(t) = b \) for \( d < t < e \) (like chain of squares).

**wave** \( x(t) \) has period=\( e-c \) and “duty cycle” \( \frac{d-c}{e-c} \) if \( a > b \), so that \( a=“\text{on}” \) and \( b=“\text{off}” \).

**stem plot** Plot of discrete-time signal using chain of vertical lines topped with circles. \( \text{stem}(X) \).

**step function** \( u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases} \) and \( u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n \leq -1 \end{cases} \). Note that \( u[0] = 1 \).

**step response** \( s[n] \): Response of a system to a step function: \( u[n] \rightarrow \boxed{\text{LTI}} \rightarrow s[n] \)

**superposition** Linear system: Response to a superposition is the superposition of the responses. Another term for **linear combination**, but applied specifically to system inputs.

**support** of \( x(t) \) is interval \([a,b]\) means that \( x(t) = 0 \) for all \( t > a \) and \( t < b \). Same for \( x[n] \).

**system function** Another name for transfer function \( H(z) \). Term used mostly in DSP.

**Time-invariance** A system is TI if there are no “n”s outside brackets. **EX:** \( y[n] = nx[n] \) is not TI.

**time scale** \( x(at) \) is \( x(t) \) compressed if \( a > 1 \); expanded if \( 0 < a < 1 \); time-reversed if \( a < 0 \).

**transfer** \( H(z) = \mathcal{Z}\{h[n]\} = \text{z-transform of impulse response} \) of an LTI system. So what?

**function** Relates: \( \text{IMPULSE} \rightarrow \text{RESPONSE} \star \text{RESPONSE} \rightarrow \text{DIFFERENCE POLE–ZERO} \rightarrow \text{EQUATION} \rightarrow \text{DIAGRAM} \)

**triangle** Periodic \( x(t) = a - |t| \) for \( |t| < a \) and \( x(t) = 0 \) rest of period (chain of triangles).

**unit circle** The circle \( \{z : |z| = 1\} = \{z : z = e^{j\omega}\} \) in the complex plane, centered at origin.

**circle** An LTI system is BIBO stable if and only if all poles are inside the unit circle.

**zeros** of a rational-function transfer-function \( H(z) = \frac{N(z)}{D(z)} \) are the roots of \( N(z) = 0 \).

**EX:** \( H(z) = \frac{z^2-3z+2}{z^2-3z+2} \rightarrow z^2 - 3z + 2 = 0 \rightarrow \{\text{zeros}\} = \{1, 2\} \). roots([1 -3 2])=1.2.

**z-xform** of \( x[n] \) is \( X(z) = \mathcal{Z}\{x[n]\} = x[0] + x[1]z^{-1} + x[2]z^{-2} + \ldots \) **EX:** \( \mathcal{Z}\{3, 1, 4\} = \frac{3z^2+z+4}{z^3} \).