

EECS 216 Winter 2008
Lab 2: FM Detector
Part I: Intro & Pre-lab Assignment
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1 Introduction

In the first few weeks of EECS 216, you learned how to determine the response of an LTI system by convolving its input with its impulse response. More recently, you have been studying the frequency domain description of signals in terms of Fourier series and Fourier transform representations, the response of LTI systems to sinusoidal inputs, the frequency transfer function of an LTI system, and the frequency domain description of linear filtering.

In Lab 2, you will characterize a number of electrical systems based on LTI concepts and tools. In particular, you will analyze, construct, and characterize an operational-amplifier circuit whose output is the derivative of its input (i.e., $y(t) \propto dx(t)/dt$). You will consider the frequency response function of this LTI system, $H(j\omega)$. You will pass periodic signals through this circuit and will compute the output signal using both time and frequency domain techniques. You will observe that both of these techniques yield identical answers, and that these answers are consistent with your measured results in lab.

As part of Lab 2, you will also analyze and characterize an *envelope detector* circuit using a pn junction diode, a resistor and a capacitor. An envelope detector can be used to recover the envelope (i.e., amplitude) of a modulated sine wave signal. In particular, you will demonstrate that a properly designed envelope detector will produce $y(t) = |A(t)|$ as its output when its input is $x(t) = A(t) \cos(\omega_c t + \phi)$. An envelope detector is a key component in AM radios. You will use an envelope detector both in this lab, and in Lab 3, where you build an AM radio.

By connecting the output of your differentiator to the input of your envelope detector you will construct a *frequency discriminator*. When the input of the frequency discriminator is $x(t) = A \cos(\omega_c t + \phi(t))$, its output is given by $y(t) \propto |\omega_c + \frac{d\phi(t)}{dt}|$. Frequency discriminators are important components in many signal processing and communication systems. They are also used in FM radios. For the present lab, you will use your frequency discriminator circuit to measure the frequency of a source.

2 Goals

- To gain a better appreciation of frequency response concepts and to apply them to increase your understanding of filtering.
- To understand both the time-domain and frequency-domain interpretations of filtering and recognize that these two interpretations, though different, describe the same system.
- To analyze, construct and characterize an operational amplifier circuit that performs differentiation on its input signal (i.e., output $\propto \frac{d}{dt}$ (input)).
- To learn about envelope detectors and to construct and analyze an envelope detector consisting of a pn junction diode, a resistor and a capacitor.
- To construct a frequency discriminator using your differentiator and your envelope detector.

- To use your frequency discriminator to measure the oscillation frequency of a source.

3 Preliminary information: Envelope Detector

3.1 The Idea of an Envelope

Consider the following voltage signal

$$v(t) = (1 + 0.5 \cos(\omega_m t)) \cos(\omega_c t) \quad (3.1.1)$$

If $\omega_m \ll \omega_c$, then the second of the two sinusoids on the right-hand side of the equation above oscillates many times before the first of the two sinusoids completes one cycle. Under these conditions, we call this “slowly varying” characteristic the *envelope* of the signal. An example is shown in Fig. 3.1.1, where the top panel shows the signal, the middle panel highlights the slow variations signal amplitude, and the bottom panel plots this slow variation as a separate function. We call ω_m and ω_c the *modulation* and *carrier* frequencies, respectively. For the case shown, the modulation frequency is 3 Hz and the carrier frequency is 80 Hz.

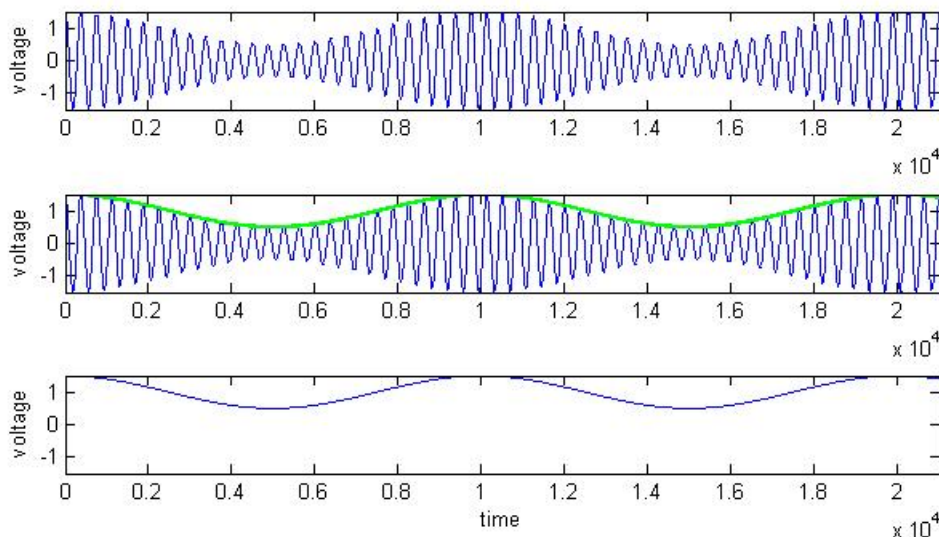


Figure 3.1.1: An envelope is a temporal feature of signals. The voltage signal in the top panel can be viewed as a sinusoid who’s amplitude is varying slowly over time. The middle panel shows this slow variation in amplitude as a second function that “rides on top of” the sinusoid. The bottom panel isolates this second function. We call the second function the *envelope* of the original signal in the top panel.

When a signal can be written in the form of Eq. 3.1.1, it is clear what is the envelope $(1 + 0.5 \cos(\omega_m t))$ and what is the carrier $\cos(\omega_c t)$. If you aren’t given a mathematical expression for the signal ahead of time, however, it is not necessarily straight forward to find a factoring of the form

$$v(t) = e(t)s(t)$$

where the *envelope*, $e(t)$, varies “slowly” with respect to some more “faster varying” *signal*, $s(t)$. Indeed, it turns out that an exact expression for a signal’s envelope requires a little more finesse and a lot more mathematical development.

Envelopes are so important in many branches of electrical and computer engineering that long before all the theory was cleaned up, practical devices were designed which performed something close to the decomposition illustrated in Fig. 3.1.1. Such devices are called *envelope detectors*. The rest of Section 3 is devoted to one such envelope detector that you will be using in Labs 2 and 3. It will be built around a nonlinear circuit component, the *diode*.

3.2 PN Junction Diode

Although this course is concerned primarily with linear time-invariant systems, we will take a short digression to discuss a nonlinear semiconductor device that will play an important role in this lab and Lab 3 (AM radio). The symbol for a pn junction diode is shown in Fig. 3.2.1 below. This device operates as a uni-directional

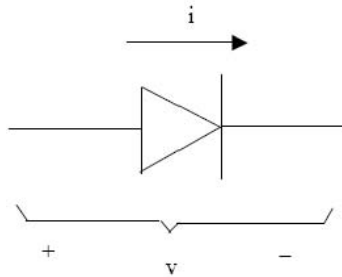


Figure 3.2.1: pn junction diode

current valve. When the diode is *forward-biased*, i.e., $v > 0$, the device lets current flow in the direction indicated above. In contrast, when the device is *reverse-biased*, i.e., $v < 0$, the current, i , flowing through the device is practically zero.

In practice, device operation deviates somewhat from this idealization. A sketch of the current vs. voltage characteristic of a common pn junction diode is shown in Fig. 3.2.2. In particular, when a small forward-biased voltage is applied, a small current flows in the forward direction. When the forward-biased voltage approaches the “turn-on” value (typically ~ 0.65 V for a silicon device and ~ 0.2 V for a germanium device) the current in the forward direction increases rapidly as shown in Fig. 3.2.2. When the device is reverse-biased, a very small amount of “leakage” current flows in the reverse direction until the breakdown voltage (typically 50 V or more) is reached. Thus, when forward-biased, the device operates as a resistor with a small resistance ($\sim < 100$ ohms); when reverse-biased, the device operates as an open circuit, allowing no current to flow in either direction.¹

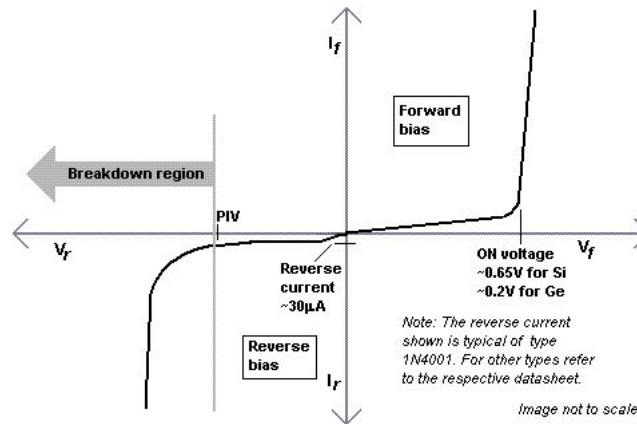


Figure 3.2.2: I-V Characteristic of a pn junction diode (from <http://en.wikipedia.org/wiki/Diode>)

¹For a detailed device datasheet for the 1N34A germanium pn junction diode that will be used in this lab see http://www.tranzistoare.ro/datasheets/70/140702_DS.pdf.

3.3 PN Junction Diode-based Envelope Detector

Consider the envelope detector circuit shown in Fig. 3.3.1. Since the behavior of the circuit changes according to the sign of the input voltage, $v_i(t)$, we consider the following two forms of the circuit.

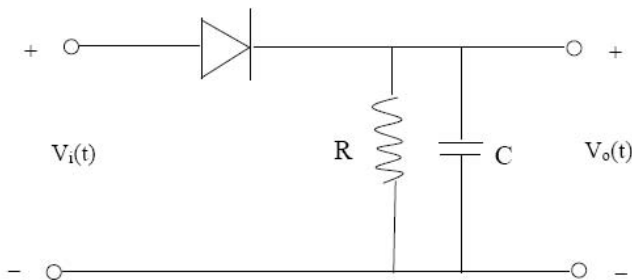


Figure 3.3.1: Envelope Detector

When the diode is forward-biased (i.e., $V_i > V_o$), the envelope detector circuit becomes the circuit shown in Fig. 3.3.2 where R_d the diode resistance is on the order of a few hundred ohms or less. It is easy to verify

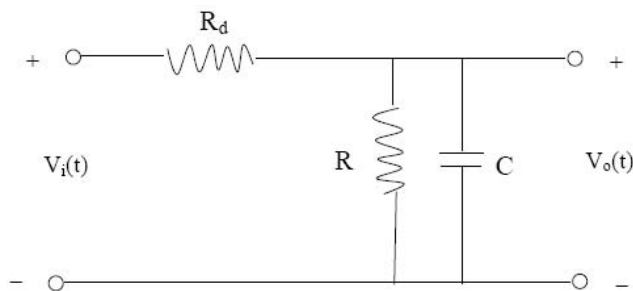


Figure 3.3.2: Forward-biased envelope detector circuit

that the forward-biased circuit has the equivalent (i.e., the same differential equation relating the input and output) representation shown in Fig. 3.3.3.

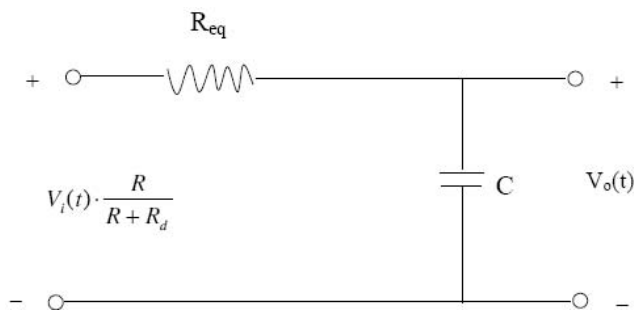


Figure 3.3.3: Forward-biased envelope detector equivalent circuit (where $R_{eq}^{-1} = R_d^{-1} + R^{-1}$ and typically $R_d \ll R$)

When the diode is reverse-biased (i.e., $V_i < V_o$) the envelope detector circuit becomes the circuit shown in Fig. 3.3.4.

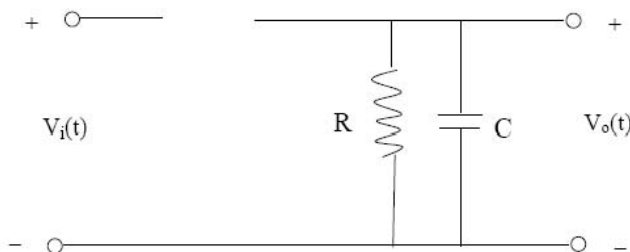


Figure 3.3.4: Reverse-biased envelope detector circuit

Given the two cases above, the behavior of the envelope-detector circuit shown in Fig. 3.3.1 with respect to a sinusoidal input voltage can be understood as follows. Let

$$V_i(t) = A(t) \sin(\omega_c t + \theta)$$

and let the following conditions be true:

- (i) $A(t) > 0$
- (ii) $\frac{1}{A(t)} \frac{dA(t)}{dt} (RC) \ll 1$
- (iii) $RC \gg \frac{2\pi}{\omega_c}$
- (iv) $R_d C \ll \frac{2\pi}{\omega_c}$ (thus $R_d C \ll RC$ and $R_{eq} \approx R_d$)

We note first that the amplitude, $A(t)$, of the sinusoid changes very slowly as the sinusoid passes through many cycles of oscillation (see conditions (ii) and (iii)). Furthermore the time-constant associated with the discharging of the capacitor (when the diode is reversed biased) is on the order of RC , while the time constant associated with the charging of the capacitor (when the diode is forward biased) is on the order of $R_{eq}C$. Thus the time required to charge is much less than the time required to discharge (see condition (iv)). In the time it would take the capacitor to either fully charge (when the diode is forward biased) or fully discharge (when the diode is reversed biased), the amplitude, $A(t)$, of the sinusoid is essentially constant (see conditions (ii), (iii) and (iv)).

Consider now the case where $\theta = 0$, $V_i(t) = A(t) \sin(\omega_c t)$ is applied at $t = t_o$, and the capacitor is initially charged to a voltage of b volts, i.e., $V_o(t_o) = bV$. According to condition (ii), the amplitude $A(t)$ can be considered essentially constant (i.e., $A(t_o)$) over a time interval of RC seconds, which corresponds to many cycles of the sinusoidal carrier (see condition (iii)). If $A(t_o)$ is less than b , the diode will be reversed biased during both the positive and negative half cycles of the sinusoid, and it will begin to discharge through the R ohm resistor. It will continue to discharge, during both positive and negative half cycles, until it reaches a voltage equal to $A(t_o)$, and the time to reach this voltage will be on the order of RC seconds. Once the capacitor voltage reaches $A(t_o)$, the small drop in voltage that occurs, due to continuous discharging during the entire duration of the negative half cycles, will be quickly and fully restored by the rapid (see condition (iv)) charging that occurs over the much smaller time interval when the diode is forward biased during the subsequent positive half cycles. Thus the output voltage will remain essentially equal to $A(t_o)$.

Similarly, suppose that $A(t_o)$ is initially greater than b , then the capacitor will fully charge over the intervals for which $A(t_o) \sin(\omega_c t)$ exceeds b during the positive half cycles of the sinusoid, reaching a voltage of $A(t_o)$ in a time on the order of $R_{eq}C$ seconds. During subsequent negative half cycles, the capacitor will discharge by a small amount that will be quickly restored during the positive half cycles.

Based on the analysis above, we see that the output of the envelope detector will, in general, be nonnegative and will tend to follow the local maxima of the signal. Fig. 3.3.5 illustrates the measured output, $V_o(t)$, of the envelope detector shown in Fig. 3.3.1 when the input is given by $V_i(t) = (2.5 + \sin(2\pi \cdot 10t)) \sin(2\pi \cdot 200t)V$, and $R = 91K\Omega$ and $C = 2.7nF$ (thus $RC = 2.7 \times 10^{-4}s$). As can be seen, the output voltage follows the local positive peaks of the signal. Examining the fine-scale behavior of the envelope detector, we see that the local

peaks of the output voltage lag slightly behind the input peaks. Once the input becomes positive, the diode is in forward-bias mode, the capacitor begins (re-)charging, and continues to charge until the input voltage crosses the origin. Once the input voltage is negative, the stored charge begins to dissipate at the much longer time constant of the reversed-bias form of the envelope detector. This fast-charge/slow-discharge behavior of the circuit is what is responsible for the scallop appearance in the fine structure of the envelope detector's output voltage.

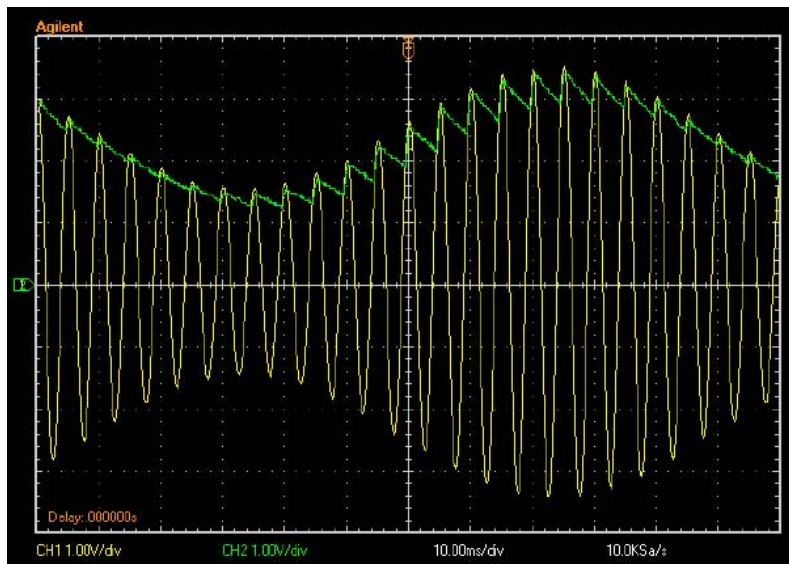


Figure 3.3.5: Envelope detector operation

If you compare Figures 3.3.1 and 3.1.1, you'll see the differences between a practical envelope detector and the ideal device it is intended to emulate. While not exact, the envelope detector does a good job of “following” the local positive peaks of the signal and interpolating between them with a “quasi-smooth” function. It also has the advantage that you can build it, it works, and it doesn't require further theoretical development! The down side, of course, is that the extracted envelope *is not* equal to the amplitude-modulation term in Eq. 3.1.1. Depending on the application, we may have to re-design the envelope detector with better “envelope tracking” circuitry or post-process the detected envelope to eliminate unwanted artifacts from the signal.

4 Pre-lab Assignment

You should work each of the problems below showing all your work. Please staple your work to the cover sheet. You may discuss the general concepts of this lab with other students in the class, but the work you turn in needs to be yours and yours alone. **You need to bring duplicate copies of your prelab homework solutions to the lab so that you can immediately compare (qualitatively), while in the laboratory, your measured results against theory. One set of your solutions will be collected at the beginning of the lab by the GSIs. Each solution should be labeled by the corresponding problem number (e.g., 4-3).**

Note: Use the Matlab axis command as specified when making plots. In addition, (i) label your plot (using the Matlab title command), (ii) label the plot axes (using the Matlab xlabel and ylabel commands), (iii) label the individual curves when multiple curves appear on a single plot (using the Matlab legend command), and (iv) use the Matlab grid command to place a fine grid on your plots. Documentation on any command can be found by typing `help "command"` in the Matlab command window (e.g., `help xlabel`).

4.1 A differentiator

- (a) Let $x(t)$ be a periodic signal of period T with the following Fourier series representation in complex exponential form:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T}kt} \quad (4.1.1)$$

Note that $\frac{dx(t)}{dt}$ is also periodic with period T . Let the Fourier series representation of $\frac{dx(t)}{dt}$ in complex exponential form be given by

$$\frac{dx(t)}{dt} = \sum_{k=-\infty}^{\infty} \hat{c}_k e^{j\frac{2\pi}{T}kt} \quad (4.1.2)$$

Express the \hat{c}_k coefficient in terms of the c_k coefficient.

- (b) Let $x(t)$ be a signal of finite energy with Fourier integral representation

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega. \quad (4.1.3)$$

Find the Fourier transform of $\frac{dx(t)}{dt}$ and express your answer in terms of $X(\omega)$.

- (c) Consider a system whose output is $\frac{dx(t)}{dt}$ whenever the input of the system is $x(t)$, where $x(t)$ is arbitrary. Note that this system is linear-time and invariant. Use your answers from part (b) above to find the frequency transfer function, $H(j\omega)$, of this system. In order to verify that your answer is correct note that

$$|H(j25)| = 25 \quad \text{and} \quad \angle H(j50) = \frac{\pi}{2}. \quad (4.1.4)$$

- (d) Use the Matlab bode, `bode(N, D, w)`, command to make magnitude and phase bode plots of $H(j\omega)$.

[Note that the frequency transfer function can be written as $H(j\omega) = \frac{N(s)}{D(s)}$, where $N(s)$ and $D(s)$ are polynomials in s with $s = j\omega$. In the Matlab bode command, `bode(N, D, w)`, the quantities **N** and **D** are row vectors that contain the polynomial coefficients of $N(s)$ and $D(s)$, respectively, listed in descending order. For example if $H(j\omega) = \frac{j\omega}{-\omega^2 + 10}$, then $N = [1 \ 0]$ and $D = [1 \ 0 \ 10]$. **w** is a vector of the frequency values (in rad/s) at which the frequency transfer function will be evaluated. Use the Matlab command `w = logspace(0, 4, 100)` which generates 100 frequency values in the range between 10^0 and 10^4 rad/s.]

4.2 Differentiator Op-Amp Circuit

- (a) Consider the operational amplifier circuit shown in Figure 4.2.1. Apply the Golden Rules of Op-Amps

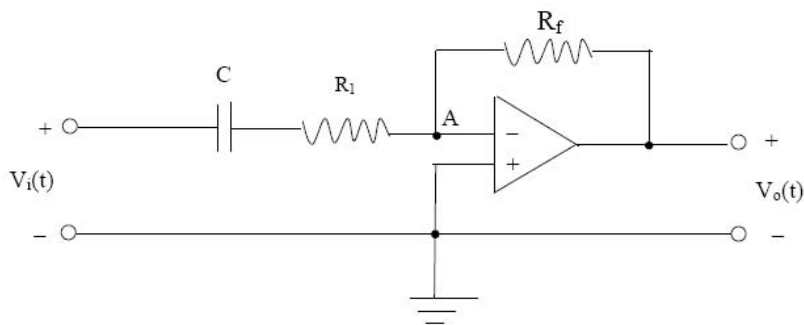


Figure 4.2.1: Operational Amplifier Differentiator Circuit

(i.e., the voltage drop across the input of the opamp is zero and the op-amp inputs draw no current) to derive the frequency transfer function, $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$, of the op-amp circuit shown in Fig. 4.2.1. (Just hand in the solution for $H(j\omega)$, not your calculations.) You may verify that your answer is correct by noting that $|H(j10^7)| = 2.28 \times 10^3$ and $\angle H(j10^7) = -1.96$ radians when $C = 2.7$ nF, $R_f = 91$ K Ω and $R_1 = 15$ Ω .

(b) Make a table of $|H(j2\pi f)|$ and $\angle H(j2\pi f)$ (in degrees) vs. f , at the following frequencies: $f = 100$ Hz, 500 Hz, 1 kHz, 2 kHz and 5 kHz when $C = 2.7$ nF, $R_f = 91$ K Ω and $R_1 = 15$ Ω .

(c) (i) Use the Matlab bode command, `bode(N,D,logspace(1,8,200))`, to plot the magnitude and phase response of this circuit when when $C = 2.7$ nF, $R_f = 91$ K Ω and $R_1 = 15$ Ω .

(ii) Use the Matlab `bode` command to plot on these *same* set of plots the magnitude and phase response of a system with frequency transfer function $\hat{H}(j\omega) = -2.457 \times 10^{-4}(j\omega)$.

Note: When the command, `bode(N,D)`, is invoked, the Matlab program automatically chooses the frequency values at which the magnitude and phase are plotted. You may specify these frequencies yourself by using the Matlab command `bode(N,D,w)`, where `w` is a row vector of the user-specified frequencies. The Matlab command `logspace(a,b,n)` generates a row vector of `n` points, each of value 10^x , where x consists of `n` equally spaced numbers starting at `a` and ending at `b`. By using the Matlab hold command (`hold`) you can issue successive plotting commands that will cause the successive sets of data to be plotted together on the same plot. The Matlab command, `holdoff`, deactivates the hold command.

(d) When $R_1 = 0$, find the frequency transfer function of this circuit and describe, in the time domain, what function this circuit performs.

(e) (i) Find the frequency transfer function, $H(j\omega)$, of a system that has the following impulse response

$$h(t) = -b \left(\frac{1}{a} \delta(t) - \frac{1}{a^2} e^{-t/a} u(t) \right). \quad (4.2.1)$$

(ii) Use convolution to compute the output, $y(t)$, of this system when the input is given by $x(t) = tu(t)$ by convolving the input with $h(t)$. In order to verify your answer note that $y(1.5a) = -0.777b$.

(iii) Evaluate your expression for $y(t)$ computed above as $a \rightarrow 0+$.

(f) Show using your part (e) answer that as $a \rightarrow 0+$ the system described by Eq. (4.2.1) performs the operation of differentiation (i.e., $y(t) = -b \frac{dx(t)}{dt}$).

(g) Suppose the input to an LTI system is $10t^2 e^{-2t} u(t)$, where the impulse response is given by (4.2.1) with $b = 0.5$. As $a \rightarrow 0+$ in the expression for the impulse response, find the output of the system. Plot your result using the Matlab axis command `axis([0 4 -2.5 1])`