EECS 216 – Winter 2008

Homework #8–Assigned March 11–Due Tuesday March 18

- **Grading:** Not all problems will be graded, but you should do all of them.
- **Submission:** Due in *black box in room 4230 EECS* **before 5:00** on Tues. March 18.
- **Read:** Text sections 4.4-4.5. **Topic:** Sampling and Fourier applications.

1. *(40 points: 8@5)* For each of these signals compute the following:
   - The minimum sampling rates and intervals to ensure no aliasing
   - The total energy of the signal
   - (a) $\frac{\sin(3\pi t)}{t} + \cos(2\pi t)$
   - (b) $\frac{\sin(\pi t)}{2t} \cos(12\pi t)$
   - (c) $[e^{-4t}u(t)] * \frac{\sin(4\pi t)}{\pi t}$
   - (d) $x(t) * [\frac{\sin(12t)}{\pi t} - \frac{\sin(8t)}{\pi t}]$
   - For (d) draw a picture and express energy in terms of $x(t)$

2. *(20 points: 5+10+5)* Are we totally hosed if aliasing occurs?
   - Let $x(t)$ have spectrum $X(\omega) = \begin{cases} X_1(\omega) & \text{for } 0 \leq |\omega| < 10\pi \\ X_2(\omega) & \text{for } 10\pi \leq |\omega| < 20\pi \\ 0 & \text{for } 20\pi \leq |\omega| \end{cases}$.
   - Find minimum sampling rate in Hertz for which we can recover from samples:
   - (a) All frequencies of the signal $x(t)$
   - (b) The low-frequency part $X_1(\omega)$ of $x(t)$
   - (c) No part of $x(t)$ without distortion

3. *(20 points: 10+10)* Inverse filtering
   - A signal $x(t)$ is processed through the differential equation
   - $\frac{d^N y}{dt^N} + a_1 \frac{d^{N-1}y}{dt^{N-1}} + a_2 \frac{d^{N-2}y}{dt^{N-2}} + \ldots + a_n y(t) = x(t)$
   - This results in blurred data $y(t)$. We wish to recover $x(t)$ from $y(t)$.
   - (a) Give a simple formula for reconstructing $x(t)$ from $y(t)$.
   - The above system is now changed to the differential equation
   - $\frac{d^N y}{dt^N} + a_1 \frac{d^{N-1}y}{dt^{N-1}} + a_2 \frac{d^{N-2}y}{dt^{N-2}} + \ldots + a_n y(t) = \frac{d^2 x}{dt^2} + 4x(t)$
   - (b) Give an example of a signal $x(t)$ that cannot be recovered from $y(t)$.

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4. *(20 points: 5+5+10)* Differentiators

- A speedometer computes $y(t) = \frac{dx}{dt}$ where $x(t)$=the odometer reading.
- White noise can be viewed as having random components at all frequencies.
- (a) Show if there is *any* white noise in the data $x(t)$, $y(t)$ is garbage.
- (b) Give $h(t)$ for a *filtered* speedometer that will work in white noise.
- (c) Does this problem arise if speed is the input and distance is the output?