

Note: $M\angle\theta = Me^{j\theta}$; angles in degrees; impedances in Ω throughout.

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1. $Y = \frac{1}{480} + j(5000)(\frac{5}{9} \times 10^{-6}) = \frac{1}{480} + \frac{j}{360} = \frac{1}{288} \angle 53^\circ$. $|I| = |V||Y| = \frac{240}{288} = \frac{5}{6}$.
 - 1a. Peak inst. power = $\frac{1}{2}|V||I| \cos(\theta_v - \theta_i) + \frac{1}{2}|V||I| = \frac{1}{2}(240)\frac{5}{6}(\cos(53^\circ) + 1) = 160W$.
 - 1b. Peak inst. power = $\frac{1}{2}|V||I| \cos(\theta_v - \theta_i) - \frac{1}{2}|V||I| = \frac{1}{2}(240)\frac{5}{6}(\cos(53^\circ) - 1) = -40W$.
 - 1c. Average power = $P = \frac{1}{2}|V||I| \cos(\theta_v - \theta_i) = \frac{1}{2}(240)\frac{5}{6} \cos(53^\circ) = 60W$.
 - 1d. Reactive power = $Q = \frac{1}{2}|V||I| \sin(\theta_v - \theta_i) = \frac{1}{2}(240)\frac{5}{6} \sin(-53^\circ) = -80var$.
 - 1e. Generates vars since $Q < 0$. **1f.** $pf = \cos(53^\circ) = 0.6$. **1g.** $rf = \sin(-53^\circ) = -0.8$.
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2. $Z = \frac{-j}{0.1} + (5||j5) + 7.5 = 10 - j7.5 = 12.5 \angle -37^\circ$. $I = \frac{50}{12.5 \angle -37^\circ} = 4 \angle 37^\circ$.
 - 2a. $S = \frac{1}{2}VI^* = \frac{1}{2}(50)(4 \angle -37^\circ) = 80 - j60VA$. $P = 80W$; $Q = -60var$; $|S| = 100VA$.
 - 2b. $I_{5\Omega} = 4 \angle 37^\circ \frac{j5}{5+j5} = 2\sqrt{2} \angle 82^\circ$. $P_{5\Omega} = \frac{1}{2}|I|^2(5\Omega) = \frac{1}{2}(2\sqrt{2})^2(5) = 20W$.
 $P_{7.5\Omega} = \frac{1}{2}|I|^2(7.5\Omega) = \frac{1}{2}4^2(7.5) = 60W$. $20 + 60 = 80W$ checks.
 - 2c. $I_{50\mu H} = 4 \angle 37^\circ \frac{5}{5+j5} = 2\sqrt{2} \angle -8^\circ$. $Q_{50\mu H} = \frac{1}{2}|I|^2(5) = \frac{1}{2}(2\sqrt{2})^2(5) = 20var$.
 $Q_{1\mu F} = \frac{1}{2}|I|^2(-10) = \frac{1}{2}4^2(-10) = -80var$. $20 - 80 = -60var$ checks.
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3. $Z = 2 + j20 + 138 + j460 = 140 + j480 = 500 \angle 74^\circ$. $I = \frac{V}{Z} = \frac{7200}{140+j480} = 14.4 \angle -74^\circ$.
 - 3a. $P_{LINE} = |I|^2R = (14.4)^2 2 = 414.72W$. Note: 7200V is rms.
 - 3b. $Y_{LOAD} = \frac{1}{138+j460} = \frac{138-j460}{230644}$. Need $j\omega C = \frac{j460}{230644} \rightarrow X_C = \frac{1}{\omega C} = \frac{230644}{460} = 501.4\Omega$.
 - 3c. $Y_{LOAD} = \frac{138}{230644} \rightarrow Z_{LOAD} = \frac{230644}{138} = 1671.3\Omega$.
 - 3d. $I = \frac{7200}{1671+j20} = 4.3 \angle -0.68^\circ$. $P_{LINE} = |I|^2R = (4.3)^2 2 = 37W$.
 - 3e. Power dissipated in line reduced from 414W to 37W: a 91% decrease! 8.9%.
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4. $pf = 0.96 = \cos(\theta_v - \theta_i) \rightarrow \sin(\theta_v - \theta_i) = \sqrt{1 - (0.96)^2} = 0.28$.
 $S_L = (25kVA)(0.96 + j0.28) = (24 + j7)kVA$. $I^* = \frac{(24+j7)kVA}{125V} = (192 + j56)A$.
 - 4a. $V_S = 125 + (192 - j56)(0.006 + j0.048) = 128.84 + j8.88 = 129.1 \angle 3.9^\circ \rightarrow |V_S| = 129.1V$.
 - 4b. $P_F = |I|^2R = |192 - j56|^2(0.006) = (200)^2(0.006) = 240W$.
 - 4c. $(125)^2(\omega C) = 7000 \rightarrow C = \frac{7000}{(125)^2 2\pi(60)} = 1188\mu F$.
 - 4d. $V_s = 125 + (192 - j0)(0.006 + j0.048) = 126.15 + j9.22 = 126.5 \angle 4.2^\circ \rightarrow |V_S| = 126.5V$.
 - 4e. $P_F = |I|^2R = |192|^2(0.006) = 221.2W$. We have reduced line loss by 19W=7.8%.
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5a. **Hard:** $Z = (4 + j\omega 0.002) || (4 - \frac{j(10)^6}{\omega 125}) = 4$ after much algebra (don't ask).

5a. **Easy:** Scale ω by 2000 (let $\xi = \omega/2000$) and use admittance, not impedance:

$$Y = \frac{1}{4+j4\xi} + \frac{1}{4-j4/\xi} = \frac{1}{4} \left(\frac{1}{1+j\xi} + \frac{j\xi}{1+j\xi} \right) = \frac{1}{4} \rightarrow Z = 4\Omega \text{ independent of } \omega \text{ (and } \xi\text{)!}$$

See how much easier this is if you *think* first before plunging into the algebra?

5b. Low frequency currents are steered into the woofer, which reproduces them well.

5b. High frequency currents are steered into the tweeter, which reproduces them well.