1a. Let $Z = R_L || \frac{1}{j\omega C} = \frac{R_L/(j\omega C)}{R_L+1/(j\omega C)} = R_L/(1 + j\omega R_L C)$.
$H(j\omega) = \frac{V_O}{V_I} = \frac{Z}{Z+R} = R_L/(R_L + R(1 + j\omega R_L C)) = \frac{1}{RC}/(j\omega + \frac{R+R_L}{RR_L C})$. 1-pole filter.
1b,c. Maximum $|H(j\omega)| = \frac{R_L}{R+R_L}$ at DC ($\omega = 0$). Check: Capacitor=open circuit at DC.
1d. $|H(j\omega)|$ down by $\frac{1}{\sqrt{2}}$=3 dB at pole frequency $\omega_c = \frac{R+R_L}{RR_L C}$.

2a,b. **(a)** $f = \frac{160{,}000}{2\pi}$=25.46kHz. **(b)** 160,000=$\frac{1}{RC} \rightarrow R = 250\Omega$.
2c. Using 1d, $\omega_c = \frac{R+R_L}{RR_L C} = \frac{1.08}{RC} \rightarrow \frac{R+R_L}{R_L} = 1.08 \rightarrow R_L = \frac{R}{0.08} = 3125\Omega$.
2d. Using 1c, $H(j0) = \frac{R_L}{R+R_L} = \frac{1}{1.08} = 0.926$. Note we don't need values of R or R_L.

3a. $\omega_o = \frac{1}{\sqrt{LC}} = \sqrt{\frac{10^3 10^{12}}{312 \cdot 1.25}} = 1.6 \times 10^6 \frac{rad}{sec}$. $f_o = \frac{\omega_o}{2\pi} = 254.8kHz$.
3b,e. $Q = \frac{\omega_o L}{R} = \frac{1.6 \cdot 312 \cdot 1000}{(50+12.5)k\Omega} = 8$. Bandwidth=$\beta = \frac{\omega_o}{Q} = \frac{1.6\times 10^6}{8} = 200k\frac{rad}{sec} = 31.83kHz$.
3c,d. $f_{c1} = f_o - \frac{\beta}{2} = 254.8 - \frac{31.83}{2} = 238.9kHz$. $f_{c2} = f_o + \frac{\beta}{2} = 254.8 + \frac{31.83}{2} = 270.7kHz$.
NOTE: Using formulae on p. 720-722, get 239.2kHz and 271.0kHz (close despite low Q).

4a. $2\pi(20kHz) = \omega_o = \frac{1}{\sqrt{LC}} \rightarrow L = \frac{10^9/10^6}{20(2\pi(20))^2} = 3.17mH$.
4a. $5 = Q = \frac{\omega_o L}{R} = \frac{40\pi 3.17}{R} \rightarrow R = \frac{40\pi 3.17}{5} = 79.58\Omega$.
4d. Bandwidth=$\beta = \frac{f_o}{Q} = \frac{20}{5} = 4kHz$. Note Q is dimensionless, so use Hz throughout.
4b,c. $f_{c1} = f_o - \frac{\beta}{2} = 20 - \frac{4}{2} = 18kHz$. $f_{c2} = f + o + \frac{\beta}{2} = 20 + \frac{4}{2} = 22kHz$.
NOTE: Using formulae on p. 720-722 (ugh), get 18.1kHz and 22.1kHz (close despite low Q).

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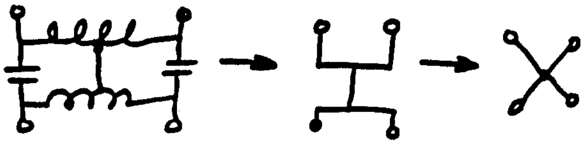
5. Bode magnitude plot starts off level, so no zero at DC ($\omega = 0$).
Up 3 dB (from 1 to 4 dB) at $\omega = 1$, so this is a zero frequency.
Levels off at 20 dB, and is down 3 dB (at 17 dB) at $\omega = 10 \rightarrow$ pole frequency.
Down 3 dB (at 17 dB) at $\omega = 100 \rightarrow$ pole frequency. No further slope changes.
$H(j\omega) = K\frac{j\omega+1}{(j\omega+10)(j\omega+100)}$. $H(j0) = 1 = K\frac{1}{(10)(100)} \rightarrow K = 1000$.

6a. *High-pass* filter$\rightarrow H(j0) = 0, H(j\infty) = 1$. See what circuit looks like at these.
$\omega = 0$: L$\rightarrow$short,C$\rightarrow$open circuit. But circuit$\rightarrow \times$, so *can't tell* (yet).
$\omega \rightarrow \infty$: L$\rightarrow$open,C$\rightarrow$short. Now circuit$\rightarrow ||$, so *input=top,output=bottom* (or vice-versa).
6b. Taking the top half of the circuit, and taking Thevenin equivalent of its left half, get
$H(j\omega) = (j\omega)^3/[(j\omega + \frac{R}{L})(-2\omega^2 + j\omega\frac{1}{RC} + \frac{1}{LC}]$ for arbitrary R,L,C.
Plugging in$\rightarrow H(j\omega) = \frac{(j\omega)^3/2}{(j\omega+a)(-\omega^2+j\omega a+a^2)}$ where $a = 3.456 \times 10^8 = 55MHz$.
Zeros: 3 at origin. **Poles:** $-a, -a\frac{1\pm\sqrt{3}}{2}$. See overleaf for more details.

NOTE: BELOW I USE $s = j\omega$ (MAKES THINGS A LITTLE NEATER)

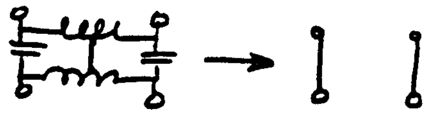
(a) KNOW FROM DESCRIPTION IT'S A HIGH-PASS FILTER, SO TRANSFER FUNCTION → 0 AT LOW FREQS. AND → 1 AT HIGH FREQS.

AT LOW FREQS: L → SHORT, C → OPEN



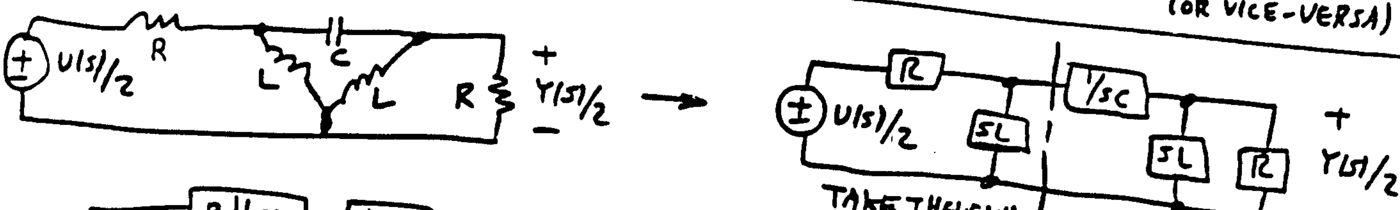
SO XFER FUNCTION IS 0, FOR ANY ORIENTATION!

AT HIGH FREQS: L → OPEN, C → SHORT

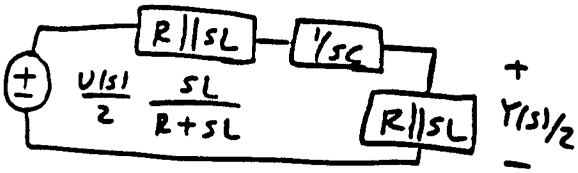


WANT XFER FUNCTION TO BE 1. OBVIOUSLY INPUT = TOP, OUTPUT = BOTTOM (OR VICE-VERSA)

(b) TAKE TOP HALF OF CIRCUIT:



TAKE THEVENIN EQUIV. OF THIS



(CAN ALSO USE CURRENT DIVIDER)

$$\to \frac{Y(s)}{2} = \frac{U(s)}{2}\left(\frac{sL}{R+sL}\right)\frac{R\|sL}{R\|sL + \frac{1}{sC} + R\|sL}$$

$$\to \frac{Y(s)}{2} = \frac{U(s)}{2}\left(\frac{sL}{R+sL}\right)\left(\frac{\frac{sLR}{sL+R}}{\frac{2sLR}{sL+R}+\frac{1}{sC}}\right)\frac{(sC)(sL+R)}{(sC)(sL+R)} = \frac{U(s)}{2}\left(\frac{sL}{R+sL}\right)\frac{(1/L)}{(1/L)}\,\frac{s^2LRC}{2s^2LRC+sL+R}\,\frac{(1/LRC)}{(1/LRC)}$$

$$= \frac{U(s)}{2}\,\frac{s^3}{(s+R/L)\left(2s^2+s\frac{1}{RC}+\frac{1}{LC}\right)}.$$

$$\therefore \boxed{\frac{Y(s)}{U(s)} = \frac{s^3}{(s+R/L)\left(2s^2+s\frac{1}{RC}+\frac{1}{LC}\right)}}$$

CHECKS: (NOT REQUIRED, BUT A GOOD IDEA!)

(1) UNITS R/L, $1/RC$, $1/LC$ ALL OK

(2) HIGH-FREQ. BEHAVIOR: $\left.\frac{Y(s)}{U(s)}\right|_{s\to\infty} = \frac{1}{2}$ (SPLIT BETWEEN THE Rs)

(3) $L\to\infty$: L = OPEN CIRCUIT. $\left.\frac{Y(s)}{U(s)}\right|_{L\to\infty} = \frac{s}{2s+1/RC}$

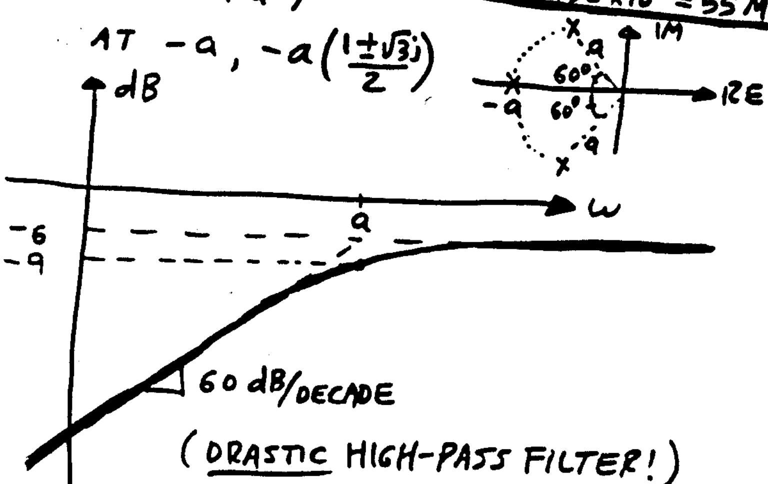
$$\to \frac{Y(s)}{U(s)} = \frac{R}{2R+1/sC} = \frac{s}{2s+1/RC}$$

(c) $R = 150$, $L = 0.434\times10^{-6}$, $C = 9.65\times10^{-12}$ →

$$\frac{s^3}{(s+3.456\times10^8)(2s^2+6.91\times10^8 s+2.388\times10^{17})}$$

$$= \frac{s^3}{2(s+a)(s^2+as+a^2)} \quad \text{WHERE } a = 3.456\times10^8 = 55\text{ MHz}$$

→ AT $-a$, $-a\left(\frac{1\pm\sqrt{3}j}{2}\right)$



(DRASTIC HIGH-PASS FILTER!)

LOW-FREQ ASYMPTOTE:
$s\to0$ GIVES $H(s) \simeq s^3/2a^3$.
AT $\omega = a$, $|H(j\omega)| = 1/2 = -6$ dB.
TRIPLE ZERO AT ORIGIN → 60 dB/DECADE

HIGH FREQ ASYMPTOTE:
$s\to\infty$ GIVES $H(s) \simeq 1/2 = -6$ dB

AT $\omega = a$: POLE AT $-a$ → DOWN 3 dB.
COMPLEX POLES $-a\left(\frac{1\pm j\sqrt{3}}{2}\right)$ →
RESONANCE PEAK HEIGHT $20\log_{10}\left(\frac{1}{2}\sec 60°\right) = 0$ dB
SUM: DOWN 3 dB.
CHECK: $\left|\left.\frac{s^3}{2(s+a)(s^2+as+a^2)}\right|_{s=ja}\right| = \frac{1}{2\sqrt{2}} = -9$ dB