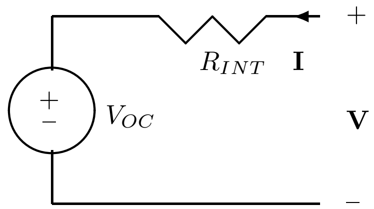
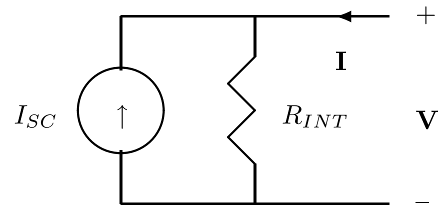
EECS 210 THEVENIN AND NORTON EQUIVALENTS Winter 2001

THM: Take *any* network of ideal voltage and current sources and resistors. Take any 2 points in the network and connect wires to them→device. Then the i-v characteristic of this device is the same as that of its:

Thevenin equivalent: **Norton equivalent:**





1. **Exception:** Pure non-zero current source has no Thevenin equivalent.
   Exception: Pure non-zero voltage source has no Norton equivalent.
2. **Why?** $V = V_{OC} + IR_{INT} \Leftrightarrow I = V/R_{INT} - I_{SC}$: Connect up I source, solve *node equations*→ V depends on I by linear+constant function.
3. **So what?** If you don't care about any *internal* voltages or current, replace circuit with its Thevenin/Norton equivalent simplifies analysis.
4. **Modelling:** We'll use these to *model* batteries & amplifier outputs.
5. **How to compute?** Do **any 2** (easiest 2) of the following 3 things:
   a. **Open-circuit voltage V_{OC}:** Simply measure voltage with $I = 0$.
   b. **Short-circuit current I_{SC}:** Clamp a short across the terminals $V = 0$ and measure the current flowing *out of* the "+" terminal.
   c. **Internal resistance R_{INT}:** First *set all* **independent** *sources to zero* (voltage sources→shorts; current sources→open (gaps)) and measure or compute the resistance between the terminals.

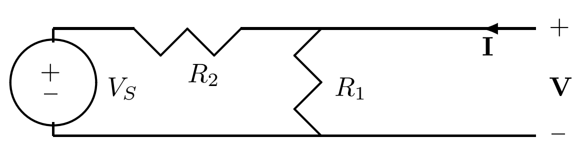
Then: $V_{THEVENIN} = V_{OC}$; $I_{NORTON} = I_{SC}$; $R_{INT} = V_{OC}/I_{SC}$.

EX#1: Compute the Thevenin and Norton equivalents of the **voltage divider:**

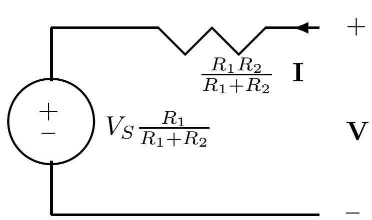
V_{OC}: $V_{OC} = V_S \frac{R_1}{R_1+R_2}$

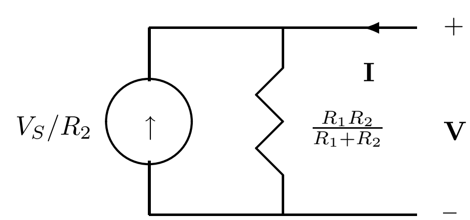
I_{SC}: $I_{SC} = V_S/R_2$

R_{INT}: $R_{INT} = R_1 || R_2$

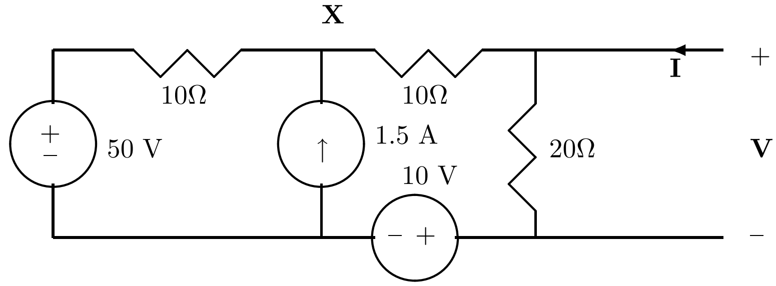


Thevenin equivalent: **Norton equivalent:**





EX#2: Compute the Thevenin and Norton equivalents of the circuit shown:



V_{OC}: Node equation at $X \rightarrow \frac{X-50}{10} + \frac{X-10}{10+20} = 1.5 \rightarrow X = 51.25V$.
Voltage divider (look carefully)$\rightarrow V_{OC} = \frac{20}{20+10}(51.25 - 10) = 27.5V$.

I_{SC}: Node$\rightarrow \frac{X-50}{10} + \frac{X-10}{10+0} = 1.5 \rightarrow X = 37.5V \rightarrow I_{SC} = \frac{37.5-10}{10} = 2.75A$.

R_{INT}: Set current source$\rightarrow 0 \Leftrightarrow$open; voltage sources$\rightarrow 0 \Leftrightarrow$short:
$R_{INT} = 20||(10+10) = 10\Omega$. **Check:** $\frac{V_{OC}}{I_{SC}} = \frac{27.5V}{2.75A} = 10\Omega = R_{INT}$.

Applications of Thevenin and Norton Equivalents:

1. A battery or stereo output can be modelled by its Thevenin equivalent: The ideal output voltage V_{OC} with an **internal resistance** R_{INT}.

EX: You test a 1.5V battery with a voltmeter and get 1.5V. But when you put it in a flashlight, it doesn't work! Now voltmeter reads 0.5V! Why?

Sol'n: Connect load$\rightarrow$*voltage division* between R_{INT} and R_L. Bulb: 5Ω.
Get $0.5 = (1.5)\frac{5}{5+R_L} \rightarrow R_L = 10\Omega$=internal resistance of battery.

Ideal: $R_{INPUT} \rightarrow \infty$ (draws no current); $R_{OUTPUT} = 0$ (R_{INT} of output).

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2. **Maximum Power Transfer⇔Load Matching:**

Given: **Fixed** Thevenin equivalent V_{OC} and R_{INT} of stereo amplifier output.
Want: **Variable** R_L (speaker). What R_L maximizes power dissipated in *load*?
Note: (R_L=0)$\rightarrow I_L^2 R_L$=0; ($R_L \rightarrow \infty$)$\rightarrow V_L^2/R_L$=0. Want intermediate R_L.

Sol'n: Power dissipated in load$=I_L^2 R_L = (\frac{V_{OC}}{R_{INT}+R_L})^2 R_L$. Minimize wrt R_L:
$\frac{d}{dR_L}[(\frac{V_{OC}}{R_{INT}+R_L})^2 R_L] = 0 \rightarrow R_L = R_{INT}$: **Match load to source**.

EX: Stereo has $R_{INT} = 8\Omega$ →use 8Ω speakers, not (say) 16Ω speakers.
Mechanical Analogue: shifting gears in car or bicycle.

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3. Circuit with *nonlinear* device: Thevenin of circuit "seen" by device.

EX: See Problem Set #5. Try solving that *without* Thevenin equivalent!