HOW TO GET POWER: Read **The Prince** by N. Machiavelli.

HOW TO DEFINE IT: Power(watts) is energy(joules) per unit time(seconds).
To move charge \( q \) "upward" across potential difference \( v \) takes energy \( E = vq \).
A charge \( q \) "falling" from higher to lower potential gains energy \( E = vq \).
This energy has to be dissipated somewhere.

If this happens continuously in time, as with **current** (flowing charge),
Power(watts) = \( \frac{dE}{dt} = \frac{dv}{dt}q = v\frac{dq}{dt} = vi \) (volts)(amps).

If potential difference caused by resistor for which \( v = iR \), Power = \( vi = i^2R = v^2/R \).
Note this is always non-negative. Makes sense: resistors dissipate power (get warm).

**Instantaneous and Average Power for Sinusoidal Voltages and Currents**

Now let \( v(t) = V_0 \cos(\omega t + \phi) \) for any \( \omega \neq 0, V_0, \phi \).
DEF: **Instantaneous power** dissipated = \( p(t) = v(t)i(t) = i^2(t)R = v^2(t)/R \).

Here \( p(t) = \frac{1}{R}V_0^2 \cos^2(\omega t + \phi) = \frac{1}{R}V_0^2 \left( 1 + \cos(2\omega t + 2\phi) \right) \).
This is a sinusoid+constant, and the frequency of the sinusoid has doubled.

DEF: **Average power** dissipated = \( \bar{p} = \frac{1}{P} \int_{t_0}^{t_0+P} p(t)dt \) where \( P = \frac{2\pi}{\omega} \) = period. Here
\[
\bar{p} = \frac{1}{P} \int_{t_0}^{t_0+P} \frac{1}{R}V_0^2 \frac{1}{2}(1 + \cos(2\omega t + 2\phi))dt = \frac{V_0^2}{2R} + \frac{V_0^2}{2R} \int_{t_0}^{t_0+P} \cos(2\omega t + 2\phi)dt = \frac{V_0^2}{2R}.
\]
IN WORDS: the average value of a constant is the constant (here \( \frac{V_0^2}{2R} \));
the average value of a sinusoid over an integer number of periods is zero.

**RMS (Root Mean Square) Voltage and Current**

Being lazy, we would like to use the same formulae \( vi = i^2R = v^2/R \)
for both **constant** \( v \) and \( i \) and **sinusoidal** \( v \) and \( i \).
We can do this by defining the **rms voltage** to be \( V_{rms} = \frac{V_0}{\sqrt{2}} \),
so that \( V_{rms} \) is the peak value of voltage divided by \( \sqrt{2} \).

Average power dissipated is \( \bar{p} = \frac{V_{rms}^2}{R} = \frac{V_0^2}{2R} \) which agrees with the correct answer.

Similarly, we define the **rms current** to be \( I_{rms} = I_0/\sqrt{2} \) where \( I_0 = \frac{V_0}{R} \),
so that \( I_{rms} \) is the peak value of current divided by \( \sqrt{2} \).
Average power dissipated is \( \bar{p} = I_{rms}^2R = I_0^2R/2 = \frac{V_0^2}{2R} \), again correct.

**EXAMPLE:** A wall socket puts out about \( v(t) = 170 \cos(2\pi 60t) \) volts.
\( V_{rms} = \frac{170}{\sqrt{2}} = 120 \) volts, which sounds familiar.
But the **peak** voltage at the wall socket is 170 volts!