EECS 210 SUMMARY OF MESH EQUATIONS Winter 2001

DEF: **Mesh:** A *loop* which does not contain any other loops inside it.
A circuit often looks like a multi-paned window; each pane is a mesh.

DEF: **Mesh current:** defined to flow around the perimeter of a mesh.

KCL: Automatically satisfied; mesh currents flow into and out of nodes.

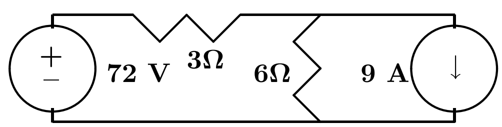
Note: Mesh analysis only works for *planar* circuits (unlike node eqns).

Note: Mesh analysis is "…probably used more often than it should be; other methods are often simpler" (Hayt and Kemmerly, 4^{th} ed., p. 67).

PROCEDURE FOR WRITING MESH EQUATIONS:

1. Define *mesh currents* $\{I_1, I_2 \ldots I_N\}$ flowing clockwise in each mesh.
   Each circuit "windowpane" should have an associated mesh current.
2. **KVL** around each mesh: sum of voltage drops around mesh is zero.
3. Each current source not on perimeter is enclosed by a *supermesh*:
   Write KVL around exterior of the two meshes that share current source.
   Difference of the two mesh currents is given by the current source.
   Still define mesh currents for each mesh; just don't write KVL for each.
4. *Dependent sources:* Express indpt variables in terms of mesh currents.
5. Solve the linear system of equations for the unknown mesh currents.
   Compute other voltages and currents of interest from mesh currents.

SIMPLE EXAMPLE

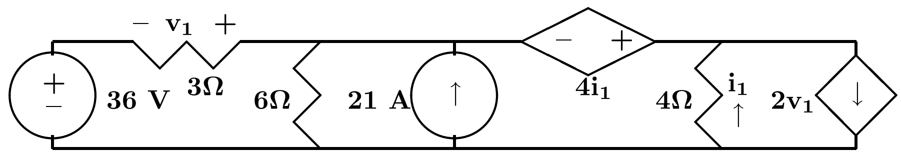


- Define *mesh current* I_1 around the left mesh; I_2 around right.
- Write KVL around left mesh: $72 - 3I_1 - 6(I_1 - I_2) = 0$.
- Current source on perimeter in right mesh: $I_2 = 9$. Get $I_1 = 14$.
- Compute other voltages and currents and check conservation of power:

ELEMENT	CURRENT	VOLTAGE	POWER
72 V :	$I_1 = 14$	$72\,(source)$	$(72)(14) = 1008$
3Ω :	$I_1 = 14$	$(14)(3) = 42$	$(42)(14) = 588$
6Ω :	$I_1 - I_2 = 5$	$(5)(6) = 30$	$(30)(05) = 150$
9 A :	$I_2 = 9$	$(5)(6) = 30$	$(30)(09) = 270$

- Power conserved: $1008 = 588 + 150 + 270$ checks.
- Note that the 9 A current source *dissipates* power (not unusual).

COMPLEX EXAMPLE



Note: This example contains all four types of sources. Shows: supermeshes; and dealing with dependent sources depending on voltage and current.

- Define *mesh currents* I_1, I_2, I_3, I_4 around meshes left to right.
- Write KVL around left mesh: $36 - 3I_1 - 6(I_1 - I_2) = 0.$
- Write KVL around supermesh consisting of middle two meshes. Supermesh encloses 21 A current source: $6(I_1 - I_2) + 4i_1 - 4(I_3 - I_4) = 0.$
- Within the supermesh, current source $\rightarrow I_3 - I_2 = 21.$
- Express indpt variables v_1 and i_1 in terms of mesh currents: $v_1 = -3I_1;\quad i_1 = I_4 - I_3;\quad 2v_1 = I_4 \rightarrow I_4 = -6I_1.$
- Substitute the second of these into the supermesh equation: $6(I_1 - I_2) + 4(I_4 - I_3) - 4(I_3 - I_4) = 0$ entirely in terms of mesh currents.
- Solve four equations in four unknowns I_1, I_2, I_3, I_4:

$$\begin{array}{rrrrl@{\qquad}rr}
9I_1 & -6I_2 & & & = 36 & I_1 = & -\frac{4}{3} \\
6I_1 & -6I_2 & -8I_3 & +8I_4 & = 00 & I_2 = & -8 \\
 & -I_2 & +I_3 & & = 21 & I_3 = & 13 \\
6I_1 & & & +I_4 & = 00 & I_4 = & 08
\end{array}$$

- Compute indpt voltages and currents from mesh currents: $v_1 = -3I_1 = 4;\quad i_1 = I_4 - I_3 = 8 - 13 = -5.$
- Compute other voltages and currents and check conservation of power:

ELEMENT	**CURRENT**	**VOLTAGE**	**POWER**
36 V :	$-I_1 = 4/3$	36 (*source*)	$(36)(4/3) = 48$
3Ω :	$-I_1 = 4/3$	$(4/3)(3) = 4$	$(4)(4/3) = 16/3$
6Ω :	$I_1 - I_2 = 20/3$	$(20/3)(6) = 40$	$(40)(20/3) = 800/3$
21 A :	21 (*source*)	$(20/3)(6) = 40$	$(40)(21) = 840$
$\mathbf{4i_1}$:	$-I_3 = -13$	$4(-5) = -20$	$(-20)(-13) = 260$
4Ω :	$I_3 - I_4 = 5$	$(4)(5) = 20$	$(20)(5) = 100$
$\mathbf{2v_1}$:	$I_4 = 8$	$(4)(5) = 20$	$(20)(8) = 160$

- Power conserved: $840 = 48 + 16/3 + 800/3 + 260 + 100 + 160$ checks.
- Note that three out of the four sources *dissipate* power (unusual).