DEF: In EECS 210 a **capacitor** is a device with i-v characteristic $i = C\frac{dv}{dt}$.

Energy: $\substack{\text{stored}\\ \text{energy}} = \int_0^t i(\tau)v(\tau)d\tau = \int_0^t Cv(\tau)\frac{dv}{d\tau}d\tau = \int_0^{v(t)} Cv\,dv = \frac{1}{2}Cv(t)^2$.

Series: Capacitors in series: $\frac{1}{C_{EQ}} = \frac{1}{C_1} + \ldots + \frac{1}{C_N}$. **N=2:** $C_{EQ} = \frac{C_1 C_2}{C_1+C_2}$.

Parallel: Capacitors in parallel: $C_{EQ} = C_1 + \ldots + C_N$ (EX: parallel plates).

Derive: $v(t) = v(0) + \frac{1}{C}\int_0^t i(\tau)d\tau$; $\quad i(t) = i(0) + \frac{1}{L}\int_0^t v(\tau)d\tau$. Also useful.

Note: Short capacitor: $-\frac{dv}{dt}$ huge$\rightarrow$ $-i(t)$ huge$\rightarrow$**spark**. EX: camera flash.

Thus: Capacitors tend to retard sharp voltage changes. EX: auto ignition.

Sinusoid: $v(t) = \cos(\omega t) \rightarrow i(t) = -\omega C\sin(\omega t)$. Gain$=\omega C$; Phase$=90^o$.

$i(t) = \cos(\omega t) \rightarrow v(t) = \frac{1}{\omega C}\sin(\omega t)$. Gain$=\frac{1}{\omega C}$; Phase$=-90^o$.

Physics: A capacitor stores **separated** charge; its *net* charge is zero. $q = Cv$.

More stored charge$\rightarrow$bigger electric field$\rightarrow$bigger potential difference.

Time-varying voltage$\rightarrow$time-varying charge$\rightarrow$current flow.

If voltage and current **constant** in time, capacitor=**open** circuit.

EX: **Parallel-plate:** $C = \frac{A\epsilon}{d}$ (ϵ=permittivity, A=area, d=separation).

Units: Farads=coulombs/volts; Joules=(coulombs)(volts)=(farads)(volts)^2.

DEF: In EECS 210 an **inductor** is a device with i-v characteristic $v = L\frac{di}{dt}$.

Energy: $\substack{\text{stored}\\ \text{energy}} = \int_0^t i(\tau)v(\tau)d\tau = \int_0^t Li(\tau)\frac{di}{d\tau}d\tau = \int_0^{i(t)} Li\,di = \frac{1}{2}Li(t)^2$.

Parallel: Inductors in parallel: $\frac{1}{L_{EQ}} = \frac{1}{L_1} + \ldots + \frac{1}{L_N}$. **N=2:** $L_{EQ} = \frac{L_1 L_2}{L_1+L_2}$.

Series: Inductors in series: $L_{EQ} = L_1+\ldots+L_N$ (EX: longer coil (more turns)).

Note: Open inductor: $-\frac{di}{dt}$ huge$\rightarrow$ $-v(t)$ huge$\rightarrow$**spark**. EX: spark plug.

Thus: Inductors tend to retard sharp current changes. EX: "choke" coils.

Sinusoid: $i(t) = \cos(\omega t) \rightarrow v(t) = -\omega L\sin(\omega t)$. Gain$=\omega L$; Phase$=90^o$.

$v(t) = \cos(\omega t) \rightarrow i(t) = \frac{1}{\omega L}\sin(\omega t)$. Gain$=\frac{1}{\omega L}$; Phase$=-90^o$.

Physics: An inductor stores *magnetic flux*=(magnetic field)(area). $\Phi = Li$.

More magnetic flux$\rightarrow$bigger magnetic field$\rightarrow$bigger current flow.

Time-varying current$\rightarrow$time-varying flux$\rightarrow$emf$\Leftrightarrow$voltage (Faraday).

If voltage and current **constant** in time, inductor=**short** circuit.

EX: **Coil:** $L = N^2\frac{A\mu}{d}$ (μ=permeability, A=area, d=length, N=#turns).

Units: Henrys=(Ohms)(sec.); Farads=(Mhos)(sec.); Joules=(henrys)(amps)^2.

Given: A circuit in which a switch is opened or closed at $t = 0$.
Where: The circuit includes: inductors; capacitors; resistors; DC sources.
Goal: What are the currents and voltages at $t = 0^-$; $t = 0^+$; $t \to \infty$?

$\mathbf{t = 0^+}$: Use following to link voltages and currents at $t = 0^-$ to $t = 0^+$:

1. Inductor *current* **can't jump:** Otherwise $V = L\frac{di}{dt} \to \infty$!
2. Capacitor *voltage* **can't jump:** Otherwise $I = C\frac{dv}{dt} \to \infty$!

$\mathbf{t \to \infty}$: Use following to compute voltages and currents as $t \to \infty$:

1. Circuit contains only DC (constant) sources→inductor=short circuit.
2. Circuit contains only DC (constant) sources→capacitor=open circuit.

EX: Electromagnet modelled by $10H$ in series with 4Ω (models coil resistance). After being closed awhile, switch is opened at $t = 0$. Compute $V_s(t)$ and $I_L(t)$.

$\mathbf{t = 0^-}$: Inductor=short→ $V_s = (200A)(16\Omega||4\Omega) = 640V$.
Current divider→ $I_L = (200A)\frac{16\Omega}{16+4\Omega} = 160A$.

$\mathbf{t = 0^+}$: Impossible! Inductor voltage→ ∞! Let $I_L(t) = (160A)e^{-t/1\,ms}, t > 0$.
Models: Switch is opened over a *finite time*; the current falls rapidly to zero.
Then: $V_s(t) = (10H)\frac{d}{dt}(160A)e^{-t/1\,ms} = -(10H)(160A)\frac{1}{0.001s}e^{-t/1\,ms}, t > 0$.

Note: Maximum voltage: $V_s(0)$=**-1.6 million volts!** Not ∞, but large!
$\mathbf{t \to \infty}$: $V_s = 0$ across inductor; $V_s = (200A)(16\Omega) = 3200V$ across source.
Also: Energy dissipated: $\frac{1}{2}LI_L^2 = \frac{1}{2}(10H)(160A)^2 = 128kJ$ (128,000 Joules!)

COMPLEX NUMBER MANIPULATIONS

Euler: $z = Me^{j\theta} = M\cos\theta + jM\sin\theta = R + jX$; $R = Re[z]; X = Im[z]$.
Note: $j = \sqrt{-1}$ (in EE, i=current!). Imaginary part is **not** itself imaginary!

Add: Add and subtract in rectangular form: $(3+j4)+(5+j12) = (8+j16)$.
Mult: Multiply & divide in polar form: $3 + j4 = 5e^{j53^o}$; $5 + j12 = 13e^{j67^o}$.
$(3+j4)(5+j12) = (3\cdot5-4\cdot12)+j(4\cdot5+3\cdot12) = -33+j56$ (hard).
$(3+j4)(5+j12) = 5e^{j53^o}13e^{j67^o} = 65e^{j120^o}$ (easy). Division easy too.

$$\frac{(a+jb)(c+jd)}{(e+jf)(g+jh)} = \sqrt{\frac{(a^2+b^2)(c^2+d^2)}{(e^2+f^2)(g^2+h^2)}}\exp j[\tan^{-1}\frac{b}{a}+\tan^{-1}\frac{d}{c}-\tan^{-1}\frac{f}{e}-\tan^{-1}\frac{h}{g}]$$