EECS 210 KIRCHHOFF'S VOLTAGE & CURRENT LAWS Winter 2001

Networks or Circuits Themselves: Need KVL and KCL

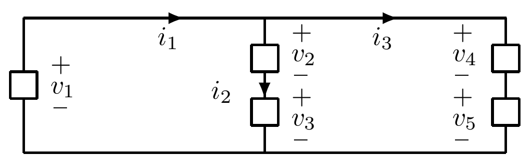
DEF: A *circuit* or *network* is an interconnection of *devices*.
Each device has a *voltage* **across** it and a *current* **through** it.

KCL: **Kirchhoff's Current Law:** Conservation of charge→
$\sum$currents flowing into a point=$\sum$currents flowing out of that point.

KVL: **Kirchhoff's Voltage Law:** Conservation of energy→
$\sum$voltages around any *closed path* in a circuit or network=0.

Why? Move a charge q around the loop. Think of a car on a rollercoaster:
Going "up" a voltage v_1, the charge *gains* potential energy qv_1.
Going "down" voltage v_2, the charge *loses* potential energy qv_2.
Charge returns to starting point→ $\sum$energy=$\sum qv_i = q \sum v_i = 0$.

KVL: $v_1 - v_2 - v_3 = 0$ (left loop)
KVL: $v_3 + v_2 - v_4 - v_5 = 0$ (right)
KVL: $v_1 - v_4 - v_5 = 0$ (outer loop)
KCL: $i_1 = i_2 + i_3$ (at both points)
Note: 1 KVL and 1 KCL redundant.

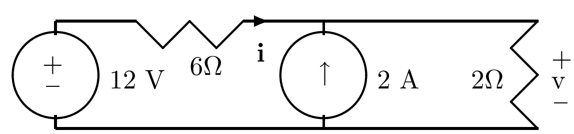


Devices in a Network or Circuit: Need i-v Characteristic

DEF: Any device can be described by rule i=function(v),
where i=current through it and v=voltage across it.
using the **standard reference directions** shown:

- (Ideal) **current source**: i=constant. EX: ideal solar cell.
- (Ideal) **voltage source**: v=constant. EX: ideal battery.
- (Ideal) **Resistor:** $v = iR$ (Ohm's law). Units: volts=(ohms)(amps).
- (Ideal) **Conductance:** $i = Gv$ where $G = 1/R$. amps=(mhos)(volts).
- (Ideal) **Short circuit:** (wire) $R = 0 \Leftrightarrow G \to \infty$.
- (Ideal) **Open circuit:** (gap) $G = 0 \Leftrightarrow R \to \infty$.

KVL: $12 - 6i - 2(i+2) = 0 \to i = 1\,A$
$\to v = 12 - 6(1) = 2(1+2) = 6\,V$



Power: Dissipated: 2Ω :(6)(1+2)=18W; 6Ω :(6)(1)=6W; Total:24W.
Power: Supplied: 2A:(6)(-2)=-12W; 12V:(12)(-1)=-12W; Total:-24W.
Check: Power dissipated=power supplied (Tellegen's law) where power=vi.

- **Voltage sources in series add:** Apply KVL
- **Current sources in parallel add:** Apply KCL
- **Resistors in series add:** $v = iR_1 + \ldots + iR_n = i(R_1 + \ldots R_n) = iR_{eq}$
- **Conductances in parallel add:** $i = G_1 v \ldots G_n v = (G_1 \ldots G_n)v$
- **Resistors in parallel: add 1/R's** (*conductances* in parallel add)
- **Voltage sources in parallel blow up!** KVL: $\sum v_i \neq 0 \rightarrow$Can't be!
- **Current sources in series blow up!** KCL: $\sum i_j \neq 0 \rightarrow$Can't be!
- **Inductors in series add:** $v = L_1 \frac{di}{dt} + \ldots + L_n \frac{di}{dt} = (L_1 + \ldots + L_n)\frac{di}{dt}$
- **Capacitors in parallel add:** $i = C_1 \frac{dv}{dt} + \ldots C_n \frac{dv}{dt} = (C_1 + \ldots + C_n)\frac{dv}{dt}$
- **Capacitors in series: add 1/C's** (see later)
- **Inductors in parallel: add 1/L's** (see later)

EX#1: $30\Omega, 60\Omega, 20\Omega$ resistors connected in series. Compute R_{eq}.
Soln: $R_{eq} = 30 + 60 + 20 = 110\Omega$.
EX#2: $30\Omega, 60\Omega, 20\Omega$ resistors connected in parallel. Compute R_{eq}.
Soln: $G_{eq} = 1/30 + 1/60 + 1/20 = 1/10$ Mhos$\rightarrow R_{eq} = 1/G_{eq} = 10\Omega$.
EX#3: N resistors of $R\Omega$ each are connected in parallel. Compute R_{eq}.
Soln: $G_{eq} = \frac{1}{R} + \ldots + \frac{1}{R} = \frac{N}{R} \rightarrow R_{eq} = \frac{R}{N}\Omega$. Parallel current paths.

Note: For **two** resistors in parallel: $R_{eq} = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2}$.
BUT: This does **not** work for more than two resistors in parallel!
$R_{eq} \neq \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3}$! Look at units: This *must* be wrong!

VOLTAGE AND CURRENT DIVIDERS

CURRENT DIVIDER

$i_1 = i_s \frac{G_1}{G_1 + G_2} = i_s \frac{R_2}{R_1 + R_2}$
$i_2 = i_s \frac{G_2}{G_1 + G_2} = i_s \frac{R_1}{R_1 + R_2}$
i_s *divided* between G_1, G_2
same voltage across both

VOLTAGE DIVIDER

$v_1 = v_s \frac{R_1}{R_1 + R_2}$ **EXS:** rheostat,
$v_2 = v_s \frac{R_2}{R_1 + R_2}$ volume control
v_s *divided* between R_1, R_2
same current through both