1. In a lab, we are measuring a periodic voltage signal with period 0.1ms.

But: 60 Hz interference from wiring in the lab walls is causing problems.

Q: Can we reduce interference by 99% using a 1-capacitor filter? How?

Specs: **Source:** Thevenin resistance=0; **Load:** input resistance=100Ω.

A: Reduce by 99% $\Leftrightarrow 40dB \Leftrightarrow$ 2 decades. $H(j\omega) = \frac{R}{\frac{1}{j\omega C}+R} = \frac{j\omega RC}{1+j\omega RC}$.

Zero at origin↔initial slope 20 dB/decade; **Pole** at $\omega = 1/(RC)$.

Use: -40 dB at 60Hz→corner freq=6kHz=$1/(2\pi(100\Omega)C) \rightarrow C = 0.27\mu F$.

Insert: C between source and load; measure output across load resistance.

Then: Fundamental and harmonics of periodic signal are hardly affected.

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2. **Tuning AM radio:** Tune in 800 kHz, reject 750 kHz by 90%.

Antenna: Antenna current=$(50\mu A)\cos(2\pi 800t) + (50\mu A)\cos(2\pi 750t)$ (t in ms).

Radio: Parallel RLC: $L = 39.6\mu H$ (loopstick antenna); C=variable; R=load.

RLC: Parallel RLC admittance: $Y = G + j\omega C + \frac{1}{j\omega L} = G + j(\omega C - \frac{1}{\omega L})$.

800 kHz: Parallel RLC *resonating:* $Y = G$. **750 kHz:** Off resonant peak.

Tune: $2\pi(800kHz) = 1/\sqrt{(39.6\mu H)C} \rightarrow$ C=1 nF. Adjust variable C to 1 nF.

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750 kHz: Set $|Y| = \sqrt{G^2 + (\omega C - \frac{1}{\omega L})^2} = 10G$ (down 90%) at $\omega = 2\pi(750kHz)$.

since: Larger Y →smaller voltage across load R for the given antenna current.

Soln: $L = 39.6\mu H$ given; C varied to 1 nF to tune in 800 kHz. Get $R = 15k\Omega$.

So: Radio amplifier should have input resistance of $15k\Omega$ (is reasonable).

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3. Series RLC has gain shown. **Plot is 3 lines through these points:**

3a. What is the form of $H(j\omega)$? **(0 db@w=0.1); (0 db@w=1);**

3b. What element(s) are tapped? **(-20 db@w=10); (-60 db@w=100);**

3c. $R = 11\Omega$. What are L and C? **(-100 db@w=1000); etc.**

3a. $H(j\omega) = 10/[(j\omega+1)(j\omega+10)]$ since: DC gain=1; Poles at $\omega = 1; 10$.

3b. DC gain=1→must tap C; Hi-frequency gain$\simeq \frac{1}{\omega^2}$ →must tap *only* C.

3c. $H(j\omega) = \frac{1/(j\omega C)}{R+j\omega L+\frac{1}{j\omega C}} = \frac{1}{(j\omega)^2(LC)+j\omega(RC)+1} = \frac{1}{(j\omega)^2(0.1)+j\omega(1.1)+1}$.

These must be equal. So we an equate coefficients of powers of $j\omega$:

$\omega^2 \rightarrow LC = 0.1; \omega \rightarrow RC = 1.1 \rightarrow L = 1H; C = 0.1F$ since $R = 11\Omega$.

Note: We can ignore phase response here, since we know we have series RLC. Phase is important in identifying unknowns in more complex problems.

Given: *Non*-ideal (finite gain) op-amp connected as an *inverting amplifier*.
Goal: Show (gain)(bandwidth)=constant (general property of amplifiers).
Note: *Lab Book* notes this in Lab Expt 4, p.9 (Post-Lab) and Unit 4, p.1.

Fact: Amplifiers tend to have a *midband* range of freqs in which gain=constant.
But: Gain rolls off for frequencies below and above this midband range.
See: *Additional Course Notes* Fig. 1.10 on p.15 and *Lab Book* Unit 3, p.9.
DEF: At *Half-power=3 dB* frequencies, gain is down 3 dB from midband gain.

Recall: For op-amp with finite open-loop gain A, inverting amplifier gain is:
$\frac{V_O}{V_I} = -\frac{R_F}{R_I}\frac{1}{1+\frac{1}{A}(1+\frac{R_F}{R_I})} \simeq -\frac{R_F}{R_I}$j as $A \to \infty$ (see *Op-Amps* handout).
Specs: Open-loop low-frequency gain:106 dB=2×10^5. Pole frequency:8 Hz.
Model: Transfer function=$A(jf) = \frac{1.6\times 10^6}{jf+8}$. Note $A(0) = \frac{1.6\times 10^6}{0+8} = 2 \times 10^5$.

Plug: $\frac{V_O}{V_I}(f) = -\frac{R_F}{R_I}/[1 + \frac{jf+8}{1.6\times 10^6}(1 + \frac{R_F}{R_I})] \approx -\frac{1.6\times 10^6}{jf+1.6\times 10^6\frac{R_I}{R_F}}$ (1-pole filter).
Using: $\frac{R_F/R_I}{1+R_F/R_I} \approx 1 \Leftrightarrow R_F/R_I >> 1$ and $1.6 \times 10^6\frac{R_I}{R_F} >> 8$ (reasonable).
Then: Closed-loop low-frequency gain=$-\frac{1.6\times 10^6}{0+1.6\times 10^6\frac{R_I}{R_F}} = -\frac{R_F}{R_I}$ as expected.
And: Pole frequency=3 dB frequency=bandwidth=$1.6 \times 10^6\frac{R_I}{R_F}$ in Hertz.

So: (Gain)(Bandwidth)=$(\frac{R_F}{R_I})(1.6 \times 10^6\frac{R_I}{R_F}) = 1.6 \times 10^6$=constant.
Means: We can trade gain for bandwidth; consider filtering of, e.g.,square wave.

ACTIVE FILTERS (OP-AMP FILTER CIRCUITS)

Why?: Can *combine* the two circuits below to produce *new* and *better* filters:
Like: Lowpass+highpass=bandpass; lowpass+lowpass=higher-order lowpass.
Also: Load at op-amp output does not affect filter; isolate input if needed.

Low-pass filter (note $\omega \to 0$)
$H(j\omega) = -\frac{R_2}{R_1}\frac{1}{j\omega R_2C+1}$

High-pass filter (note $\omega \to \infty$)
$H(j\omega) = -\frac{R_2}{R_1}\frac{j\omega R_1C}{j\omega R_1C+1}$