1. In a lab, we are measuring a periodic voltage signal with period 0.1ms. **But:** 60 Hz interference from wiring in the lab walls is causing problems. **Q:** Can we reduce interference by 99% using a 1-capacitor filter? How? **Specs:** **Source:** Thevenin resistance=0; **Load:** input resistance=100Ω.

**A:** Reduce by 99% \(\Leftrightarrow 40\, \text{dB} \Leftrightarrow 2\, \text{decades}.
\[
H(j\omega) = \frac{R}{j\omega C + R} = \frac{j\omega RC}{1 + j\omega RC}.
\]
**Zero** at origin \(\Rightarrow\) initial slope 20 dB/decade; **Pole** at \(\omega = 1/(RC)\).

**Use:** -40 dB at 60Hz \(\rightarrow\) corner freq=6kHz=1/(2\pi(100\Omega)C) \(\rightarrow\) \(C = 0.27\mu F\).

**Insert:** C between source and load; measure output across load resistance. **Then:** Fundamental and harmonics of periodic signal are hardly affected.

2. **Tuning AM radio:** Tune in 800 kHz, reject 750 kHz by 90%.

**Antenna:** Antenna current\(=\)\((50\mu A) \cos(2\pi 800t) + (50\mu A) \cos(2\pi 750t)\) (t in ms).

**Radio:** Parallel RLC: \(L = 39.6\mu H\) (loopstick antenna); \(C=\text{variable}\); \(R=\text{load}\).

**RLC:** Parallel RLC admittance: 
\[
Y = G + j\omega C + \frac{1}{j\omega L} = G + j(\omega C - \frac{1}{\omega L}).
\]

800 kHz: Parallel RLC resonating: \(Y = G\). **750 kHz:** Off resonant peak.

Tune: \(2\pi(800\, \text{kHz}) = 1/\sqrt{(39.6\mu H)C} \rightarrow C=1\, \text{nF}\). Adjust variable C to 1 nF.

750 kHz: Set \(|Y| = \sqrt{G^2 + (\omega C - \frac{1}{\omega L})^2} = 10G\) (down 90%) at \(\omega = 2\pi(750\, \text{kHz})\). **since:** Larger \(Y \rightarrow\) smaller voltage across load R for the given antenna current.

**Soln:** \(L = 39.6\mu H\) given; \(C\) varied to 1 nF to tune in 800 kHz. Get \(R = 15k\Omega\).

**So:** Radio amplifier should have input resistance of 15k\Omega (is reasonable).

3. Series RLC has gain shown. **Plot is 3 lines through these points:**

<table>
<thead>
<tr>
<th>3a</th>
<th>What is the form of (H(j\omega))? (0 db@w=0.1); (0 db@w=1);</th>
</tr>
</thead>
<tbody>
<tr>
<td>3b</td>
<td>What element(s) are tapped? (-20 db@w=10); (-60 db@w=100);</td>
</tr>
<tr>
<td>3c</td>
<td>(R = 11\Omega). What are L and C? (-100 db@w=1000); etc.</td>
</tr>
</tbody>
</table>

3a. \(H(j\omega) = \frac{10}/[(j\omega + 1)(j\omega + 10)]\) since: DC gain=1; Poles at \(\omega = 1; 10\).

3b. DC gain=1 \(\Rightarrow\) must tap C; Hi-frequency gain\(\approx \frac{1}{\omega^2} \rightarrow\) must tap only C.

3c. \(H(j\omega) = \frac{1}{R+j\omega L+\frac{1}{j\omega C}} = \frac{1}{(j\omega)^2(LC)+j\omega(RC)+1} = \frac{1}{(j\omega)^2(0.1)+j\omega(1.1)+1}.
\]

These must be equal. So we an equate coefficients of powers of \(j\omega:\)
\(\omega^2 \rightarrow LC = 0.1; \omega \rightarrow RC = 1.1 \rightarrow L = 1H; C = 0.1F\) since \(R = 11\Omega\).

**Note:** We can ignore phase response here, since we know we have series RLC. Phase is important in identifying unknowns in more complex problems.
Given: Non-ideal (finite gain) op-amp connected as an inverting amplifier.

Goal: Show \((\text{gain})(\text{bandwidth})=\text{constant}\) (general property of amplifiers).

Note: Lab Book notes this in Lab Expt 4, p.9 (Post-Lab) and Unit 4, p.1.

Fact: Amplifiers tend to have a midband range of freqs in which gain=constant.
But: Gain rolls off for frequencies below and above this midband range.
See: Additional Course Notes Fig. 1.10 on p.15 and Lab Book Unit 3, p.9.
DEF: At Half-power=3 dB frequencies, gain is down 3 dB from midband gain.

Recall: For op-amp with finite open-loop gain \(A\), inverting amplifier gain is:
\[
\frac{V_O}{V_I} = -\frac{R_F}{R_I} \left[ \frac{1}{1 + \frac{R_F}{R_I}} \right] \approx -\frac{R_F}{R_I} \quad \text{as} \quad A \to \infty \quad \text{(see Op-Amps handout)}.
\]

Specs: Open-loop low-frequency gain:106 dB=\(2 \times 10^5\). Pole frequency:8 Hz.
Model: Transfer function\(=A(j\omega) = \frac{1.6 \times 10^6}{j\omega + 8}\). Note \(A(0) = 1.6 \times 10^6 = 2 \times 10^5\).

Plug: \[
\frac{V_O}{V_I}(f) = -\frac{R_F}{R_I}/\left[ 1 + \frac{j\omega + 8}{1.6 \times 10^6}(1 + \frac{R_F}{R_I}) \right] \approx -\frac{1.6 \times 10^6}{j\omega + 1.6 \times 10^6 \frac{R_F}{R_I}} \quad \text{(1-pole filter)}.
\]

Using: \(\frac{R_F/R_I}{1 + R_F/R_I} \approx 1 \iff R_F/R_I >> 1 \quad \text{and} \quad 1.6 \times 10^6 \frac{R_F}{R_I} >> 8 \quad \text{(reasonable)}\).
Then: Closed-loop low-frequency gain\(=-\frac{1.6 \times 10^6}{0 + 1.6 \times 10^6 \frac{R_I}{R_F}} = -\frac{R_F}{R_I}\) as expected.
And: Pole frequency=3 dB frequency=bandwidth=\(1.6 \times 10^6 \frac{R_I}{R_F}\) in Hertz.

So: \((\text{Gain})(\text{Bandwidth})=\left(\frac{R_F}{R_I}\right)(1.6 \times 10^6 \frac{R_I}{R_F}) = 1.6 \times 10^6=\text{constant}\).

Means: We can trade gain for bandwidth; consider filtering of, e.g., square wave.

**ACTIVE FILTERS (OP-AMP FILTER CIRCUITS)**

Why?: Can combine the two circuits below to produce new and better filters:
Like: Lowpass+highpass=bandpass; lowpass+lowpass=higher-order lowpass.
Also: Load at op-amp output does not affect filter; isolate input if needed.

Low-pass filter (note \(\omega \to 0\)) \[
H(j\omega) = -\frac{R_2}{R_1} \frac{1}{j\omega R_2 C + 1}
\]

High-pass filter (note \(\omega \to \infty\)) \[
H(j\omega) = -\frac{R_2}{R_1} \frac{j\omega R_1 C}{j\omega R_1 C + 1}
\]