Given: Transfer function factored into form $H(j\omega) = \frac{(j\omega - z_1)\ldots(j\omega - z_M)}{(j\omega - p_1)\ldots(j\omega - p_N)}$.

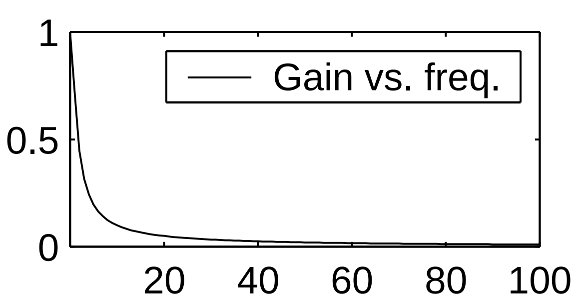
Goal: Plot *gain* $|H(j\omega)|$ and *phase* $\angle H(j\omega)$ vs. frequency $\omega = 2\pi f$ on a *log-log scale*: $\log|H(j2\pi f)|$ vs. $\log(f)$ $\Leftrightarrow$express gain in **decibels**.

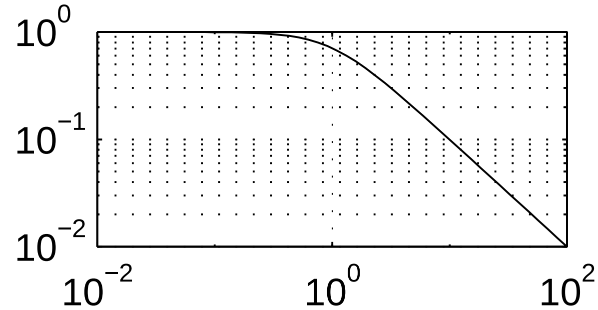
EX: Voltage source v_S with internal resistance R connected to capacitor C.
Input: $v_S(t) = \cos(2\pi f t)$. **Output:** $v_C(t) = M\cos(2\pi f t + \theta)$.
Phasors: $V_C = V_S \frac{1/(j\omega C)}{R + 1/(j\omega C)} = V_S \frac{1}{1 + j\omega RC}$ (voltage divider)

Then: Transfer function$=H(j\omega) = \frac{V_C}{V_S} = \frac{1}{1+j\omega RC} = \frac{1}{\sqrt{1+(\omega RC)^2}}\angle -\tan^{-1}(\omega RC)$.

Plot: **Gain**$=\frac{1}{\sqrt{1+(2\pi f RC)^2}}$ vs. f; **Phase**$=-\tan^{-1}(2\pi f RC)$ vs. f.

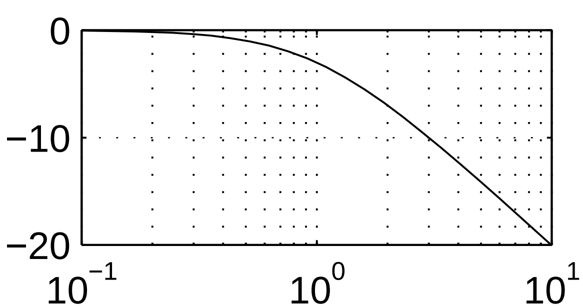
Gain: Plot gain vs. frequency on regular plot and log-log plot, for $RC = \frac{1}{2\pi}$:

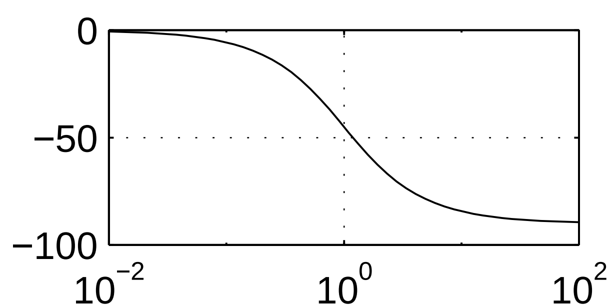




Note: Advantages of using a log-log plot, instead of a regular plot:
1. Plot consists of two straight lines connected by a short curve;
2. Can depict gain values over a much broader range of frequencies;
3. Can extend easily to more complex transfer functions (see below).

Idea: Plot $20\log_{10}|H(j2\pi f)|$=gain in **decibels** vs. frequency on log scale:





Note: Following features of Bode gain and phase plots on semilog scales:
1. Gain plot lines have slopes of zero or -6db/octave=-20db/decade, where octave=factor of 2 and decade=factor of 10 and $6 \approx 20\log_{10} 2$.
2. Sloped line intercepts 0 dB at "corner frequency" $f = 1/(2\pi RC)$ Hz.
3. AT the corner frequency, gain$=\frac{1}{\sqrt{2}} = -3$ dB and phase=-45 degrees.

Given: Transfer function $H(j\omega) = 10\frac{-\omega^2+j\omega(10^6+10^4)+10^{10}}{-\omega^2+j\omega(10^8+100)+10^{10}} = 10\frac{(j\omega+10^4)(j\omega+10^6)}{(j\omega+100)(j\omega+10^8)}$.

Goal: Plot Bode magnitude plot for this transfer function.

Idea: $20\log_{10}|H(j\omega)| = 20\log_{10}(10) + 20\log_{10}|j\omega+10^4| + 20\log_{10}|j\omega+10^6|$ $-20\log_{10}|j\omega+100| - 20\log_{10}|j\omega+10^8|$ since dB of products add.

DEF: *Zeros* are roots of numerator polynomial of $H(j\omega)$ (z_i overleaf).

DEF: *Poles* are roots of denominator polynomial of $H(j\omega)$ (p_i overleaf).

Point: We can handle each individual term as we did overleaf. Procedure:

1. In $H(j\omega)$, let $\omega \to 0$ and compute the DC gain $|H(j0)|$ in dB.
2. At each *zero* frequency of $H(j\omega)$, *add* slope 20 dB/decade=6 dB/octave.
3. At each *pole* frequency of $H(j\omega)$, *subtract* 20 dB/decade=6 dB/octave.

The four smaller plots add together (since decibels) to give the larger plot.