
USEFUL MATHEMATICAL METHODS AND FORMULAE

Useful Trigonometry Facts and Identities

1. $\sin(0) = \cos(\frac{\pi}{2}) = 0$; $\cos(0) = \sin(\frac{\pi}{2}) = 1$; $\cos(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$.
 2. $\cos(-x) = \cos(x)$ (even function); $\sin(x) = -\sin(-x)$ (odd function).
 3. $\sin(x) = \cos(x - \frac{\pi}{2})$; $\cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x)$; $\sin^2(x) + \cos^2(x) = 1$.
 4. $\cos(x) \cos(y) = \frac{1}{2} \cos(x + y) + \frac{1}{2} \cos(x - y)$ (note $x = y \rightarrow$ above).
-

5. $e^{jx} = \cos(x) + j \sin(x)$ makes rest of trig obsolete. For example:
 $e^{jx} e^{jy} = e^{j(x+y)} = \cos(x + y) + j \sin(x + y)$. But we also have:
 $e^{jx} e^{jy} = [\cos(x) + j \sin(x)][\cos(y) + j \sin(y)]$. Multiplying out:
 $= [\cos(x) \cos(y) - \sin(x) \sin(y)] + j[\sin(x) \cos(y) + \cos(x) \sin(y)]$.
 Equating real and imaginary parts \rightarrow cos and sin addition formulae.
-

Means of Signals with Infinite Duration

EX: $x(t) = e^{-|t|}$ has support $(-\infty, \infty)$ and duration $\rightarrow \infty$.

Then: $M(x) = \frac{1}{\infty} \int_{-\infty}^{\infty} e^{-|t|} dt$. Huh? Now what do I do?

What: Compute the *Cauchy Principal Value* of this indefinite integral:

To Do: Compute $M(x)$ for finite support $[-T, T]$ and let $T \rightarrow \infty$:

Soln: $M(x) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T e^{-|t|} dt = \lim_{T \rightarrow \infty} \frac{2}{2T} \int_0^T e^{-t} dt$ (using symmetry)
 $M(x) = \lim_{T \rightarrow \infty} \frac{1}{T} (1 - e^{-T}) = 0$.

Periodic: If $x(t) = x(t + T)$ for all t (NOT just for, say, $t > 0$)

$$M(x) = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt \text{ (use 1 period).}$$

Gaps: $x(t) = \begin{cases} t & \text{for } 0 < t < 1; \\ 2 & \text{for } 2 < t < 3; \\ 0 & \text{for otherwise} \end{cases} \rightarrow \begin{aligned} E(x) &= \int_0^1 t^2 dt + \int_1^2 0^2 dt + \int_2^3 2^2 dt = \frac{13}{3}. \\ MS(x) &= E(x)/\text{duration}(x) = \frac{13/3}{3-0} = \frac{13}{9}. \end{aligned}$

Useful Formulae for Discrete-Time Signals

1. $1 + r + r^2 + \dots + r^{N-1} = \frac{1-r^N}{1-r}$ for any r .
2. $1 + r + r^2 + \dots = \frac{1}{1-r}$ if $|r| < 1$.
3. For signals having infinite support $[0, \infty)$, use
 $M(x) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} x[n]$ (compare to Cauchy Principal Value).
4. $x[n] = A \cos(\omega n + \theta)$ is periodic only if $\omega = 2\pi \left(\frac{\text{RATIONAL}}{\text{NUMBER}} \right)$.
5. $|x| < a$ is equivalent to $-a < x < a$.