CONTINUOUS AND DISCRETE TIME SIGNALS; A/D AND D/A

- **Continuous-time** signals: $x(t)$, t real. EX: speech, seismogram.
- **Discrete-time** signals: $x[n]$, n integer. EX: stock market closes.

A/D: **Analog-to-Digital** conversion: $x[n] = x(t = n\Delta)$: **Sampling**.
D/A: **Digital-to-Analog** conversion: $x(t) = \sum x[n]\phi(t - n\Delta)$: Interpolate
D/A: Do this for some basis function $\phi(t)$ (to be determined).

QUICK MATLAB DEMOS

```
load handel;soundsc(y);plot(y);z=y(1:2:length(y));soundsc(z)
N=1:8192; X=cos(N); soundsc(X); Y=cos(2*N); soundsc(Y)
Z=cos(N/1000);plot(Z);W=cos(2*N/1000);plot(W) (subplot)
M=2:8192;soundsc(y(M)+y(M-1)) (crude lowpass filtering)
```

BASIC SIGNAL PROPERTIES AND DEFINITIONS

Support: $[t_1, t_2] \Leftrightarrow x(t) \neq 0$ **only for** $t_1 \leq t \leq t_2 \Leftrightarrow x(t) = 0$ for $t < t_1$ or $t > t_2$.

Duration: $x(t)$ has support $[t_1, t_2] \Leftrightarrow x(t)$ has **duration** $t_2 - t_1$.

Duration: $x[n]$ has support $[N_1, N_2] \Leftrightarrow x[n]$ has **duration** $N_2 - N_1 + 1$!

Min;Max: $MIN(x(t))$=minimum value of $x(t)$; $MAX(x(t))$=its maximum value.

Mean: $M(x) = \bar{x}(t) = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x(t)dt$. **Discrete time:** $\frac{1}{N_2 - N_1 + 1}\sum_{n=N_1}^{N_2} x[n]$.

Periodic: $x(t) = x(t + T)$ for all $t \rightarrow M(x) = \frac{1}{T}\int_{t_o}^{t_o+T} x(t)dt$ for any t_o

MEAN SQUARE: $MS(x) = \overline{x^2}(t) = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x^2(t)dt$. **rms:** $RMS(x) = \sqrt{MS(x)}$.

Variance: $\sigma_x^2 = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}(x(t) - M(x))^2 dt = MS(x - M(x)) = MS(x) - (M(x))^2$.

STANDARD DEVIATION: $\sigma_x = \sqrt{\sigma_x^2}$. Use $\sigma_x^2 = MS(x) - (M(x))^2$ for easier computation.

Note: **Energy**=E(x)=$\int_{t_1}^{t_2} x^2(t)dt$. **Avg. Power**=MS(x)=$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x^2(t)dt$.

Units: Power=$\frac{\textbf{ENERGY}}{\textbf{TIME}}$. EX: Watts=$\frac{\textbf{JOULES}}{\textbf{SECONDS}}$. $x^2(t)$=instantaneous power.

EXAMPLE: BASIC SIGNAL PROPERTIES

GOAL: $x(t) = \begin{cases} 2t & \text{for } 0 \leq t \leq 3; \\ 0 & \text{otherwise} \end{cases}$. Compute above quantities for this signal.

Support=[0,3]. **Duration**=3-0=3. **Min**=0. **Max**=6.

Mean=$\frac{1}{3-0}\int_0^3 2t\,dt = 3$. **Mean square**=$\frac{1}{3}\int_0^3 (2t)^2 dt = 12$.

Variance=$\frac{1}{3}\int_0^3 (2t - 3)^2 dt$ (hard) $= 12 - (3)^2 = 3$ (easy).

STANDARD DEVIATION $= \sqrt{3}$. **rms**=Root-Mean-Square=$\sqrt{12}$. **Energy**=36.

SIGNAL HISTOGRAMS

Given: Discrete-time signal $x[n]$ with finite support$=[N_1, N_2]$.
If given continuous-time signal $x(t)$, **sample** it: $x[n] = x(t = n\Delta)$.

Set-up: Horizontal axis runs from $MIN(x[n])$ to $MAX(x[n])$ (range of $x[n]$).

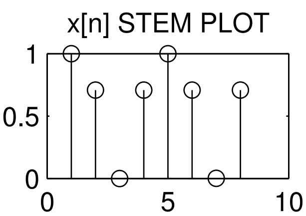
Set-up: Horizontal axis divided up into M **bins**, for some integer M.

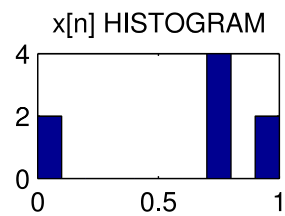
Bins: Each of M bins has width $\frac{1}{M}(MAX(x[n]) - MIN(x[n]))$.

Set-up: Vertical axis is a bar graph: **#values of $x[n]$ in that bin**.

EXAMPLE: SIGNAL HISTOGRAM

Given: $x[n] = \begin{cases} |\cos(\frac{\pi}{4}n)| & \text{for } 0 \leq n \leq 7; \\ 0 & \text{otherwise} \end{cases}$.





Goal: Plot histogram using $M = 10$ bins.

Values: $x[n]$ takes on only these 3 values:
0(twice); $\sqrt{2}/2$(4 times); 1(twice).

So what? Can estimate mean, variance, etc. from histogram: don't need $x[n]$.

How? Approximate all values of $x[n]$ in a bin by center value of that bin.

EX #1: $M(x) \approx \frac{1}{8}(2(0.05) + 4(0.75) + 2(0.95)) = 0.6125$.

Whereas: $M(x) = \frac{1}{8}(1 + \frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2}) = 0.6035$.

EX #2: $MS(x) \approx \frac{1}{8}(2(0.05)^2 + 4(0.75)^2 + 2(0.95)^2) = 0.507$.

Whereas: $MS(x) = \frac{1}{8}(1^2 + (\frac{\sqrt{2}}{2})^2 + 0^2 + (\frac{\sqrt{2}}{2})^2 + 1^2 + (\frac{\sqrt{2}}{2})^2 + 0^2 + (\frac{\sqrt{2}}{2})^2) = 0.5$.

SIGNAL SIMILARITY MEASURE: CORRELATION

DEF: The **Correlation** between $x(t)$ and $y(t)$ is $C(x, y) = \int_{t_1}^{t_2} x(t)y(t)dt$.

WHY? **Cauchy-Schwarz** inequality: $|C(x, y)| \leq \sqrt{E(x)}\sqrt{E(y)}$. That is:
$(\int_{t_1}^{t_2} x(t)y(t)dt)^2 \leq (\int_{t_1}^{t_2} x^2(t)dt)(\int_{t_1}^{t_2} y^2(t)dt)$. Derivation: Don't ask.

OR: **Correlation coefficient** $\rho(x, y)$: $|\rho(x, y)| = |\frac{C(x,y)}{\sqrt{E(x)E(y)}}| \leq 1$.

SO? The closer this is to one, the more alike are $x(t)$ and $y(t)$. Indeed,
$C(x, y) = \pm\sqrt{E(x)E(y)} \Leftrightarrow x(t) = \pm a y(t)$ for some constant $a > 0$.

SIGNAL SIMILARITY APPLICATION: TIME DELAY

Given: Observe $y(t) = x(t - D) + n(t)$ where $n(t)$=unknown noise.

Signal: $x(t)$=known signal with small support (i.e., a short pulse).

Goal: Estimate unknown time delay D from noisy data $\{y(t)\}$.

How? **Correlate** $y(t)$ with various **delayed** (by τ) versions of $x(t)$:

Compute: $C(x(t - \tau), y(t)) = \int x(t - \tau)y(t)dt$. View this as a function of τ.

Then: Look for a sharp peak at $\tau = D$ in $C(x(t - \tau), y(t))$ as a function of τ.

Note: See an example of this later in my lecture notes under "Applications."