RECURSIVE COMPUTATION OF IMPULSE RESPONSE

Goal: Compute impulse response $h[n]$ of system $y[n] - \frac{1}{2}y[n-1] = 3x[n]$.

Sol'n: Compute recursively $h[n] - \frac{1}{2}h[n-1] = 3\delta[n] = 0$ if $n > 0$.

n=0: $h[0] - \frac{1}{2}h[-1] = 3\delta[0] \rightarrow h[0] - \frac{1}{2}(0) = 3(1) \rightarrow h[0] = 3$.

n=1: $h[1] - \frac{1}{2}h[0] = 3\delta[1] \rightarrow h[1] - \frac{1}{2}(3) = 3(0) \rightarrow h[0] = \frac{3}{2}$.

n=2: $h[2] - \frac{1}{2}h[1] = 3\delta[2] \rightarrow h[2] - \frac{1}{2}(\frac{3}{2}) = 3(0) \rightarrow h[0] = \frac{3}{4}$.

n=3: $h[3] - \frac{1}{2}h[2] = 3\delta[3] \rightarrow h[3] - \frac{1}{2}(\frac{3}{4}) = 3(0) \rightarrow h[0] = \frac{3}{8}$.

$h[n] = 3(\frac{1}{2})^n u[n] = 3(\frac{1}{2})^n$ for $n \geq 0$. *Geometric* signal.

BIBO (BOUNDED INPUT→BOUNDED OUTPUT) STABILITY

Goal: Determine whether an LTI system is BIBO stable from its $h[n]$.

EX #1: $h[n] = \{2, 3, -4\} \rightarrow \sum |h[n]| = |2| + |3| + |-4| = 9 < \infty \rightarrow$ BIBO stable.

EX #2: $h[n] = (-\frac{1}{2})^n u[n] \rightarrow \sum |h[n]| = \sum |-\frac{1}{2}|^n = \frac{1}{1-0.5} < \infty \rightarrow$ BIBO stable.

EX #3: $h[n] = \frac{(-1)^n}{n+1}u[n] \rightarrow \sum |h[n]| = \sum \frac{1}{n+1}u[n] \rightarrow \infty \rightarrow$ NOT BIBO stable.

Note: $\sum \frac{(-1)^n}{n+1}u[n] = \log 2$ but $\sum |\frac{(-1)^n}{n+1}|u[n] = \sum \frac{1}{n+1}u[n] \rightarrow \infty$

so *absolute* summability vs. summability matters for BIBO stability!

CONVOLUTION OF TWO FINITE SIGNALS

Goal: Compute $\{\underline{1}, 2, 3\} * \{\underline{4}, 5, 6, 7\} = \{4, 13, 28, 34, 32, 21\}$.

$y[0] = h[0]x[0] = (1)(4) = 04$.

$y[1] = h[1]x[0] + h[0]x[1] = (2)(4) + (1)(5) = 13$.

$y[2] = h[2]x[0] + h[1]x[1] + h[0]x[2] = (3)(4) + (2)(5) + (1)(6) = 28$.

$y[3] = h[2]x[1] + h[1]x[2] + h[0]x[3] = (3)(5) + (2)(6) + (1)(7) = 34$.

$y[4] = h[2]x[2] + h[1]x[3] = (3)(6) + (2)(7) = 32$.

$y[5] = h[2]x[3] = (3)(7) = 21$.

Note: Length $y[n]$=Length $h[n]$+Length $x[n] - 1 = 3 + 4 - 1 = 6$.

CONVOLUTION OF FINITE AND INFINITE SIGNALS

Goal: Compute $\{\underline{2}, -1, 3\} * (\frac{1}{2})^n u[n] = 2\delta[n] + 3(\frac{1}{2})^{n-2}u[n-2]$.

Sol'n: $\{\underline{2}, -1, 3\} * (\frac{1}{2})^n u[n] = (2\delta[n] - 1\delta[n-1] + 3\delta[n-2]) * (\frac{1}{2})^n u[n] =$

$2(\frac{1}{2})^n u[n] - 1(\frac{1}{2})^{n-1}u[n-1] + 3(\frac{1}{2})^{n-2}u[n-2] = 2\delta[n] + 3(\frac{1}{2})^{n-2}u[n-2]$.

Note: $\{\underline{2}, -1\} * (\frac{1}{2})^n u[n] = 2\delta[n]$. So $x[n] \rightarrow \boxed{(\frac{1}{2})^n u[n]} \rightarrow \boxed{\{2, -1\}} \rightarrow 2x[n]$.