LOW-PASS FILTER DESIGN USING POLES AND ZEROS

Given: Sum of two continuous-time signals: Call this sum $x_1(t) + x_2(t)$.
such Signal $x_1(t)$ is bandlimited to 000-250 Hz (low pass signal).
that Signal $x_2(t)$ is bandlimited to 250-500 Hz (highpass signal).

Given: A DSP system having a sampling rate=1 kHz=1000 $\frac{\text{SAMPLE}}{\text{SECOND}}$.
and: A DSP chip having a total of 10 delays (storage registers).
Goal: Design a digital lowpass filter to pass $x_1(t)$ and reject $x_2(t)$.

Freqs:

contin. :	1000 Hz	500 Hz	250 Hz
discrete :	2π	π	$\pi/2$
identity :	sampling	Nyquist	cutoff

$$H(\omega) = \begin{cases} 1 & \text{for } 0 \le |\omega| < \pi/2 \\ 0 & \text{for } \pi/2 < |\omega| \le \pi \end{cases}.$$

Ideal: $h[n] = \sin(\frac{\pi}{2}n)/(\pi n)$ (sinc function). Impulse response of IIR filter.
Problems: (1) Noncausal (2) Unstable (3) No difference equation implements.

Zeros: $\{e^{\pm j\pi/2}, e^{\pm j3\pi/4}, e^{j\pi}\}$ to reject high frequencies (zeros *on* unit circle).
Poles: $\{.6, .8e^{\pm j\pi/4}, .8e^{\pm j\pi/2}\}$ to pass low frequencies (.6 works better here).
Note: Need all the poles inside the unit circle ($|p_n| < 1$) for BIBO stability.

H(z): $H(z) = \frac{(z-e^{j\pi/2})(z-e^{-j\pi/2})(z-e^{j3\pi/4})(z-e^{-j3\pi/4})(z-e^{j\pi})}{(z-.8e^{j\pi/2})(z-.8e^{-j\pi/2})(z-.8e^{j\pi/4})(z-.8e^{-j\pi/4})(z-.6)}$.

using: $(z-e^{j\pi/2})(z-e^{-j\pi/2})(z-e^{j\pi}) = (z-j)(z+j)(z+1) = z^3+z^2+z+1$
and: $(z-e^{j3\pi/4})(z-e^{-j3\pi/4}) = z^2 - 2\cos(\frac{3\pi}{4})z + 1 = z^2 + \sqrt{2}z + 1$
and: Performing similar computations for the denominator factors.

H(z): $H(z) = \frac{z^5+2.414z^4+3.414z^3+3.414z^2+2.414z+1}{z^5-1.73z^4+1.96z^3-1.49z^2+0.84z-0.25} = \frac{Y(z)}{X(z)}$. Cross-multiply:

$$y[n] - 1.73y[n-1] + 1.96y[n-2] - 1.49y[n-3] + 0.84y[n-4] - 0.25y[n-5]$$
$$= x[n] + 2.414x[n-1] + 3.414x[n-2] + 3.414x[n-3] + 2.414x[n-4] + x[n-5]$$

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  Z=[ exp(j*pi/2),exp(-j*pi/2),exp(3j*pi/4),exp(-3j*pi/4),exp(j*pi)]
P=[.8* exp(j*pi/2),.8*exp(-j*pi/2),.8*exp(j*pi/4),.8*exp(-j*pi/4),.6]
  B= poly(Z);A=poly(P);[H,W]=freqz(B,A);plot(W,abs(H)),zplane(B,A)
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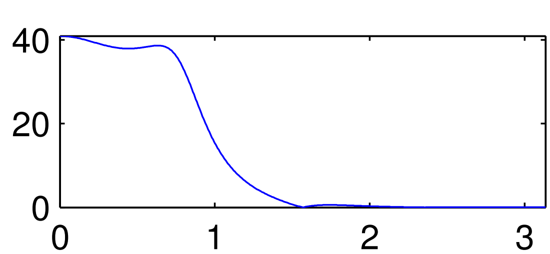

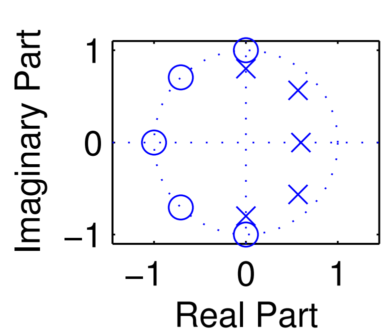

COMB FILTER DESIGN USING POLES AND ZEROS

Given: A 48 Hz sinusoidal signal plus interference (from a motor) at 60 Hz.
But: Interference *not* sinusoidal, just periodic: 4 harmonics up to 240 Hz.

Given: DSP system having: (1) sampling rate of 480 Hz; and (2) 14 delays.
Goal: Filter out 60 Hz periodic interference, while keeping 48 Hz sinusoid.

DSP: $x(t) \rightarrow$ | ANTI–ALIAS | $\rightarrow$ | SAMPLE @480Hz | $\rightarrow$ | COMB FILTER | $\rightarrow$ | INTERP–OLATOR | $\rightarrow y(t)$

x[n]: Discretized sinusoidal signal has $\omega = 2\pi \frac{48\,Hz}{480\,Hz} = 0.2\pi$. Want to keep.
x[n]: Interference has $\omega = 2\pi \frac{60\,Hz}{480\,Hz} = 0.25\pi$ and harmonics at $0.5\pi, 0.75\pi, \pi$.

Goal: Reject $\omega = 0.25\pi, 0.5\pi, 0.75\pi, \pi$ from interference; keep 0.2π sinusoid.
Zeros: $\{e^{\pm j\pi/4}, e^{\pm j\pi/2}, e^{\pm j3\pi/4}, e^{j\pi}\}$ to reject harmonics of the interference.
Poles: $\{.9e^{\pm j\pi/4}, .9e^{\pm j\pi/2}, .9e^{\pm j3\pi/4}, .9e^{j\pi}\}$ to keep all the other frequencies.
Note: Poles inside unit circle ($|p_n| < 1$) for BIBO stability. $h[n] \approx (.9)^n u[n]$.

H(z): $$\frac{(z-e^{j\pi/4})(z-e^{-j\pi/4})(z-e^{j\pi/2})(z-e^{-j\pi/2})(z-e^{j3\pi/4})(z-e^{-j3\pi/4})(z-e^{j\pi})}{(z-.9e^{j\pi/4})(z-.9e^{-j\pi/4})(z-.9e^{j\pi/2})(z-.9e^{-j\pi/2})(z-.9e^{j3\pi/4})(z-.9e^{-j3\pi/4})(z-.9e^{j\pi})}$$

using: $(z - e^{j\pi/4}) \ldots (z - e^{j8\pi/4}) = z^8 - 1$ (the eight 8^{th} roots of unity)
and: $(z - e^{j\pi/4}) \ldots (z - e^{j7\pi/4}) = \frac{z^8-1}{z-1} = z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z + 1$

H(z): $\frac{z^7+z^6+z^5+z^4+z^3+z^2+z+1}{z^7+.9z^6+.9^2z^5+.9^3z^4+.9^4z^3+.9^5z^2+.9^6z+.9^7} = H(z) = \frac{Y(z)}{X(z)}$. Cross-multiply:

difference equation $y[n] + (.9)y[n-1] + (.9)^2 y[n-2] + \ldots + (.9)^6 y[n-6] + (.9)^7 y[n-7]$
$= x[n] + x[n-1] + x[n-2] + \ldots + x[n-5] + x[n-6] + x[n-7]$

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E=exp(j*pi/4);Z=[E,E^2,E^3,E^4,E^5,E^6,E^7];
P=.9*Z; N=0:79; S=[3,1,4,1,5,-9,2,-7]/2; 0-mean interference
X=1+cos(.2*pi*N)+[S S S S S S S S S S];M=60:79; repeat period
subplot(421),stem(M,X(M)); B=poly(Z);A=poly(P); start at 60
subplot(424),zplane(B,A); [H,W]=freqz(B,A); to allow transient
subplot(422),plot(W,abs(H)); Y=filter(B,A,X); to decay to 0
subplot(423),stem(M,real(Y(M))); Interference rejected
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