
LINE SPECTRUM OF SIGNAL=SUM OF SINUSOIDS

Given: $x(t) = A_0 + A_1 \cos(\omega_1 t + \theta_1) + \dots + A_N \cos(\omega_N t + \theta_N)$.

Sum of N sinusoids at N **different frequencies**.

Goal: Represent amplitudes and phases as scaled impulses vs. frequency.

Using $Re[z] = \frac{1}{2}(z + z^*)$ with $z = X_k e^{j\omega_k t}$, we have:

- $x(t) = A_0 + \frac{1}{2} \sum_{k=1}^N (X_k e^{j\omega_k t} + X_k^* e^{-j\omega_k t})$ where $X_k = A_k e^{j\theta_k}$.

1. Can represent a sinusoid at frequency $\omega_k = 2\pi f_k$ by *two complex exponentials* at frequencies $\pm\omega_k$.
 2. Note need negative frequencies; watch amplitude factor of two.
 3. Note component at negative frequency $-\omega_k$ is complex conjugate of its counterpart component at positive frequency ω_k .
 4. Plot: impulses=lines in frequencies; hence *line spectrum*.
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EX: $x(t) = 10 + 14 \cos(2t - 1) + 8 \cos(4t + 3)$ has 5 complex components: (10 at $\omega = 0$); ($7e^{\mp j}$ at $\omega = \pm 2$); ($4e^{\pm j3}$ at $\omega = \pm 4$) (see plot overleaf).

BEAT SIGNALS (SOUND LIKE THEY ARE BEATING)

What: Identity $\cos(\omega_1 t + \theta_1) + \cos(\omega_2 t + \theta_2) = 2 \cos(\omega_c t + \theta_c) \cos(\omega_\Delta t + \theta_\Delta)$.

where: $\omega_c = \frac{1}{2}(\omega_1 + \omega_2)$ and $\omega_\Delta = \frac{1}{2}(\omega_1 - \omega_2)$; θ_c, θ_Δ are defined similarly.

Why? Apply cosine addition formula and $\omega_1 = \omega_c + \omega_\Delta$ and $\omega_2 = \omega_c - \omega_\Delta$.

Easier: Note that $2 \cos(\omega_c t + \theta_c) 2 \cos(\omega_\Delta t + \theta_\Delta)$ can be written as product $[e^{j(\omega_c t + \theta_c)} + e^{-j(\omega_c t + \theta_c)}][e^{j(\omega_\Delta t + \theta_\Delta)} + e^{-j(\omega_\Delta t + \theta_\Delta)}]$.

Multiply this out \rightarrow 4 terms $\rightarrow 2 \cos(\omega_1 t + \theta_1) + 2 \cos(\omega_2 t + \theta_2)$ (try it!).

Sounds $x(t)$ sounds like a single tone at frequency $\omega_c \approx \omega_1, \omega_2$

like: whose *amplitude varies slowly with time* at frequency ω_Δ .

EX #1: Tuning a piano. We wish to tune a piano key to 440 Hz.

Given: A tuning fork tuned to A above middle C, which is 440 Hz.

Given: Piano mistuned to 441 Hz. Want to change this to 440 Hz.

Sol'n: Strike tuning fork and piano key simultaneously and listen.

$$\rightarrow \text{Hear } \cos(2\pi 440t) + \cos(2\pi 441t) = 2 \cos(2\pi 0.5t) \cos(2\pi 440.5t)$$

Sounds A single tone at 440.5 Hz whose amplitude varies at 0.5 Hz.

like: Actually 1 Hz since amplitude is non-negative. Period=1 second.

Sol'n: Adjust piano key until amplitude variation ceases (Period $\rightarrow \infty$).

EX #2: AM radio. WJR (760 kHz) transmits a tone (EBS?) at 5 kHz.

$$\begin{aligned} x(t) &= 4 \cos(2\pi(760,000)t) \cos(2\pi(5000)t) \\ &= 2 \cos(2\pi(755,000)t) + 2 \cos(2\pi(765,000)t) \\ &= e^{j2\pi(765,000)t} + e^{j2\pi(755,000)t} + e^{-j2\pi(765,000)t} + e^{-j2\pi(755,000)t} \end{aligned}$$

Line spectrum: 4 components in two closely-spaced pairs.

